Introduction to Industrial Organization
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Lecture Note 12

Price discrimination (ch 10)-continue

2nd degree price discrimination

We have discussed that firms charged different prices to consumers with different characteristics. Called 1st and 3rd price discrimination. The whole purpose of this is to charge higher prices to consumers with higher willingness to pay.

What if consumer 1 has $WTP_1>WTP_2$ where WTP_2 is 2's willingness to pay? And there are no observable differences between 1 and 2? i.e. the firm cannot offer consumer 2 a discount for being a senior citizen.

If the firm sets p1=WTP_1 and p2=WTP_2 and asks the consumers to reveal their identities, consumer 1 will claim to be #2 in order to pay a lower price.

This class we will explain how firms engage in price discrimination when consumers have <u>private</u> information about their willingness to pay for a product. The firm's goal is the same: make a sale to a low WTP consumer without having to lower prices charged to high WTP consumers. But now, firm has to get consumers to "admit" their preferences. It does this by offering different types of products (large vs small, high quality vs low quality) and pricing carefully…

One way that firms do this is through quantity discounting: low per unit price when larger quantities purchased.

An example of quantity discounting is drink sizes and prices. At Starbuck's:

Small coffee is 12 oz and costs \$1.75 ➔ \$0.15/ounce

Large coffee is 20 oz and costs \$1.95 ➔ \$0.010/ounce

What we can show that this pricing policy is an attempt at 2nd degree price discrimination.
Consider demand curve for an arbitrary cup of coffee.

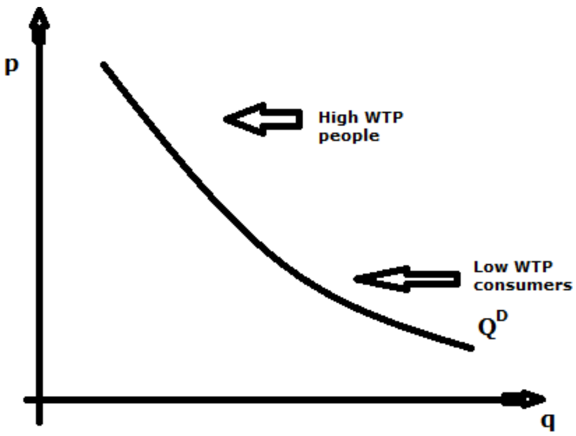

Figure 10.e4 Demand for coffee

In the figure above, there are two types of consumers, high WTP and low WTP. The monopolist would like to be able to charge the high WTP consumers a high price and the low WTP consumers a low price. That way the firm can extract lots of surplus from the high types and still be able to sell to the low types.

Assume that the monopolist knows that there are high WTP consumers and low WTP consumers, but the monopolist is unable to distinguish between the two types, i.e. 1st and 3rd price discrimination is not possible.

Instead the monopolist will engage in 2nd degree price discrimination. He will design two products: a large cup of coffee with a high price. This product will be aimed at high WTP consumers. The high price will enable the monopolist to extract lots of surplus from high WTP consumers. And a small cup of coffee with a low price aimed at the low WTP consumers. The low price will ensure that they purchase coffee.

Setup:

- A cup of coffee has a size (q ounces) and a price (p)
- Consumers set untility $u = \theta \ln(q) - p$ where θ reflects consumers WTP for extra coffee. $\frac{du}{dq} = \frac{\theta}{q}$ so high θ consumers have a high MU from an extra ounce.
- There are two types of consumers, θ_H *and* θ_L, with $\theta_H > \theta_L$.
- Starbucks wants to engage in 2nd degree price discrimination. Will offer two cups of coffee. A large (q_H, p_H) aimed at θ_H and a small (q_L, p_L) aimed at θ_L.
- Q ounces of coffee costs Starbucks cq dollars.

Starbuck's problem: design two cups of coffee such that

θ_H purchases (q_H, p_H), θ_L purchases (q_L, p_L) and profits are maximized.

$$\pi_{Starbucks} = \max_{q_H, q_H, q_L, q_L} \{p_H - cq_H + p_L - cq_L\}$$

Such that

$$(1)\ \theta_H \ln(q_H) - p_H \geq 0$$
$$(2)\ \theta_L \ln(q_L) - p_L \geq 0$$

$$(3)\ \ \theta_H \ln(q_H) - p_H \geq \theta_L \ln(q_L) - p_L$$
$$(4)\ \ \theta_L \ln(q_L) - p_L \geq \theta_L \ln(q_H) - p_H$$

Constraints (1) and (2): θ_H and θ_L prefers to buy coffee instead of buying nothing.

Constraints (3) and (4): consumers buy the product designed for them. θ_H prefers (q_H, p_H) to (q_L, p_L) and θ_L prefers (q_L, p_L) to (q_H, p_H).

This problem can be simplified:

Claim (1): Constraint (1) can be ignored.

- θ_H consumers must get positive utility from (q_L, p_L) since $\theta_H > \theta_L$ and $\theta_L q_L - q_L \geq 0\ implies\ (\theta_H q_L, p_L) > 0$.
- Therefore θ_H must get positive utility from (q_H, p_H) because of constraint (3)

$$\theta_H \ln(q_H) - p_H \geq \theta_H \ln(q_L) - p_L > 0$$

Claim 2: Constraint (2) is binding.

We know that $\theta_H \ln(q_H) - p_H \geq 0$. If it is also true that $\theta_L \ln(q_L) - p_L \geq 0$ then Starbucks is not maximizing profits. Can increase p_H and p_L by a tiny amount, increase profits and all of the constraints are still satisfied.

Starbuck's problem is now:

$$\pi = \max(p_H - cq_H + p_L - cq_L)$$

Such that

$$(2)\ \ \theta_L \ln(q_L) - p_L = 0$$
$$(3)\ \ \theta_H \ln(q_H) - p_H \geq \theta_H \ln(q_L) - p_L$$
$$(4)\ \theta_L \ln(q_L) - p_L \geq \theta_L \ln(q_H) - p_H$$

Claim 3: Constraint (3) is binding.

If it is not binding then $\theta_H \ln(q_H) - p_H > \theta_H \ln(q_L) - p_L$. Starbucks can decrease q_H by a little bit, increase profits, and constraints 92) and (4) still hold.

Claim 4: We can ignore constraint (4).

If constraint (3) is binding, then $\theta_H \ln(q_H) - p_H = \theta_H \ln(q_L) - p_L$ or $\theta_H(\ln(q_H) - \ln(q_L)) = p_H - p_L$. Since $\theta_L < \theta_H$, $\theta_L(\ln(q_H) - \ln(q_L)) < p_H - p_L$ or $\theta_L \ln(q_L) - p_L > \theta_L \ln(q_H) - p_H$

So, Starbuck's problem becomes:

$$\pi = \max(p_H - cq_H + p_L - cq_L)$$

Such that

$$\theta_L \ln(q_L) - p_L = 0$$
$$\theta_H \ln(q_H) - p_H = \theta_L \ln(q_L) - p_L$$

Intuition: Starbuck's wants to design a low quality and a high quality product. θ_L is just indifferent between purchasing the low quality product and nothing. The monopolist extracts all of their surplus. The monopolist must design the high quality product so that θ_H sets positive utility because otherwise θ_H will purchase the low quality product. But to maximize profits, the high quality product is designed so that θ_H is indifferent between the two products.

Now we just plug the constraints into the profit function and solve:

$$\max_{q,p} (p_H - cq_H + p_L - cq_L)$$

Such that

$$\theta_L \ln(q_L) - p_L = 0 \Leftrightarrow p_L = \theta_L \ln(q_L)$$

$$\downarrow$$

$$\theta_H \ln(q_H) - p_H = \theta_H \ln(q_L) - p_L \Leftrightarrow p_H = \theta_H[\ln(q_H) - \ln(q_L)] + \theta_L \ln(q_L)$$

$$\Rightarrow$$

$$\max_{q_H, q_L} (q_H \ln(q_H) - \theta_H \ln(q_L) + \theta_L \ln(q_L) - cq_H + \theta_L \ln(q_L) - cq_L)$$

$$\begin{cases} \dfrac{d\pi}{dq_H} = \dfrac{\theta_H}{q_H} - c = 0 \\ \dfrac{d\pi}{dq_L} = \dfrac{2\theta_L - \theta_H}{q_L} - c = 0 \end{cases}$$

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$$\begin{cases} q_H^* = \dfrac{\theta_H}{c} \\ q_L^* = \dfrac{2\theta_L - \theta_H}{c} \\ p_H^* = \theta_H[ln(q_H^*) - \ln(q_L^*)] + \theta_L \ln(q_L^*) \\ p_L^* = \theta_L \ln(q_L^*) \end{cases}$$

We observe a Starbuck's menu:

$$q_L = 12\ oz$$
$$p_L = \$1.75$$
$$q_H = 20\ oz$$
$$p_H = \$1.95$$

Can our model duplicate this?

If $c = 0.04$, $\theta_L = 0.647$, and $\theta_H = 0.804$, our model predicts $q_L^* = 12.26$, $p_L^* = 1.62$, $q_H^* = 20.1$, $p_H^* = 2.02$

Our model is not perfect because of utility assumptions, restricting to 2 sizes instead of 3, but can do a pretty good job of duplicating Starbuck's pricing pattern.

The 2nd degree price discrimination was able to explain Starbuck's nonlinear pricing policy as an attempt to extract surplus from high WTP consumers while still making sales to low WTP consumers.

The Starbuck's model was one of quantity discounting. We can redefine "q" and explain lots of other real world phenomenon:

1. q= quality
   - electronics companies offer two versions of the same product, one with extra feature (q_H), the other basic one (q_L)
   - insurance companies offer menus of insurance plans, some with low coverage (q_L) others with high coverage
2. q= rewards programs
   - restaurants offer deals: buy 10 sandwiches, get your 11th free. Here q_L=3 sandwiches over a given period of time q_H=10 sandwiches +1 free over same period of time. Per unit price is lower q_H than q_L. Pricing policy sets people who eat out for lunch a lot to always come to your restaurant.
   - Same thing for airlines, hotel reward/loyalty programs
3. q=time
   - some people really value having the newest products. Other people do not care. Charge a high price for a product immediately after it is introduced. Then lower price slowly.
   - Airlines: business travelers have higher WTP and also make plans later than other travelers. Airlines charge lower prices for tourists when ticket is for date far in the future.

In the previous discussion on price discrimination, consumers either purchase a good or they don't. For many goods, consumers buy multiple units (example: cell phone minutes. Consumers enroll in a plan, and then choose how many minutes to consume). For these types of products firms often use two part tariffs to price discriminate/extract surplus from consumers.

Two part tariff: A firm charge a consumer a fee (tariff 1) for the right to buy as many units of the product as the consumer wants at a specific price (tariff 2).

Examples:

- Cell phone plans
- membership discount retailers: Costco
- amusement parks

The price a consumer pays for

$$q \text{ units} = F + pq$$
where F is tariff 1
p is tariff 2

This is nonlinear pricing since each unit cost $(\frac{F}{q} + p)$ depends on q.

First, let's show that a monopolist can use two part tariffs to extract more surplus from consumers.

Suppose a consumer demands $q^D(p)$ units of output if the monopolist charges a price p.

Figure 10.e5 Demand curve for one individual

The monopolist can either use a one part tariff $(F = 0, p \geq 0)$ or a two part tariff $(F \geq 0, p \geq 0)$. Assume the monopolist has constant marginal costs c.

1) monopolist uses a 1 part tariff.

$$\pi = \max_{p} \{p_q(p) - cq(p)\}$$

Just like standard monopoly problem. Set MR=MC.

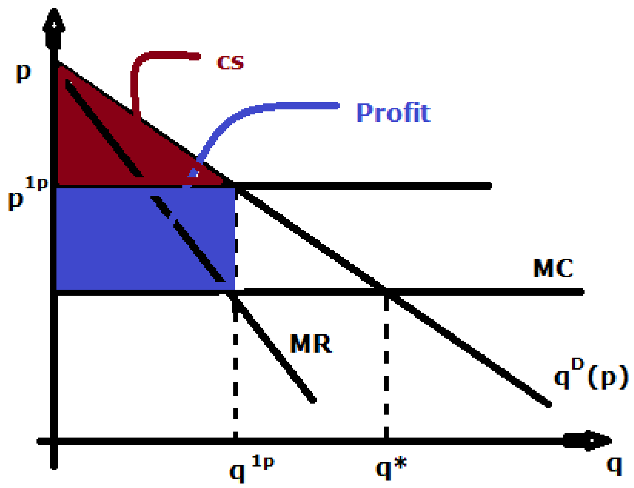

Figure 10.e6 Monopoly pricing-1 part tariff

When monopolist force to set $F = 0$, he charges a markup $(p > c)$ and there is too little output produced $q^{1p} < q^*$ and DWL>0.

2) Monopolist uses a 2 part tariff
   - Now the monopolist has two instruments it can use (F and p) to extract surplus from consumers.
   - Surplus is maximized at q^*. If the monopolist is able to choose a (F, p) so that profits=total surplus at q^*, this must be the profit maximizing solution.

Suppose the monopolist sets p=c. then the consumer will demand q^* units of output and surplus is maximized. Now, the monopolist will choose F to extract as much surplus as possible from the consumer:

Figure 10.e7 shows that if monopolist sets p=c, surplus is maximized. If monopolist then sets F=total surplus (red shaded area), consumers will still purchase the good and firms extracts all surplus.

Optimal two part tariff:

Set p=MC to get efficient allocation

Set F=Area between demand curve and price to extract all surplus from consumers

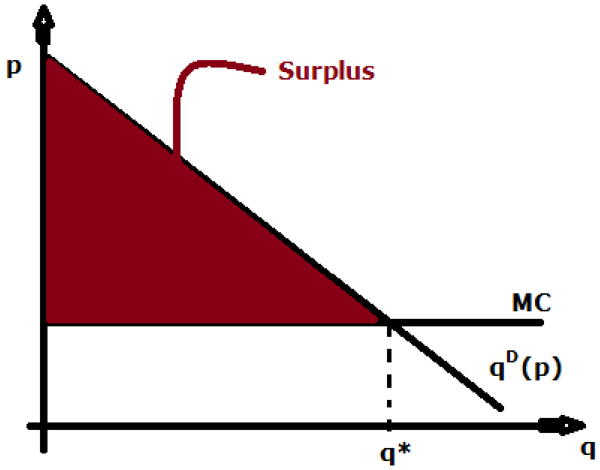

Figure 10.e7 Monopoly pricing:
F=total surplus, p=MC

This analysis explains many real pricing policies. For example, gym memberships usually have a monthly fee and unlimited free visits to the gym. This reflects the fact that the marginal cost of a gym visit=0.

Are two part tariffs good? We see a familiar trade off. Two part tariffs yield the efficient outcome since firms don't need to use high per unit prices to extract surplus on the first few units of q that are sold. But on the other hand, firms collect all of the surplus.

Now assume that there are two types of consumers.

Type H consumers value output a lot.
Type L consumers value output a little.

For example $U(q) = \theta \ln(q) - p_q - F$

$$\frac{dU}{dq} = \frac{\theta}{q} - p = 0 \rightarrow q^*(p) = \frac{\theta}{p}$$

$$p = \frac{\theta}{q}$$

Let $\theta_H > \theta_L$. Then the high θ consumers' demand curve lies on top of the low θ consumers' demand curve (in Figure 10.e8 below):

Similarly, we can lot indifference curves.

$$MU_q = \frac{\theta}{q}$$

$$MU_{other\ \ comsumption} = 1$$

Since utility linear in price.

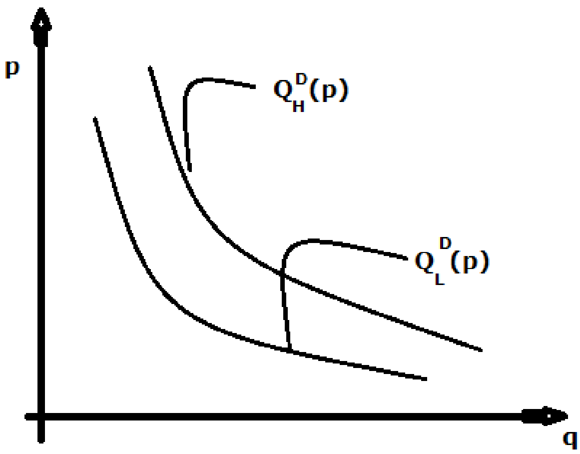

Figure 10.e8: Demand curves for the types of consumers

Therefore $\text{MRS} = -\frac{\theta}{q}$.

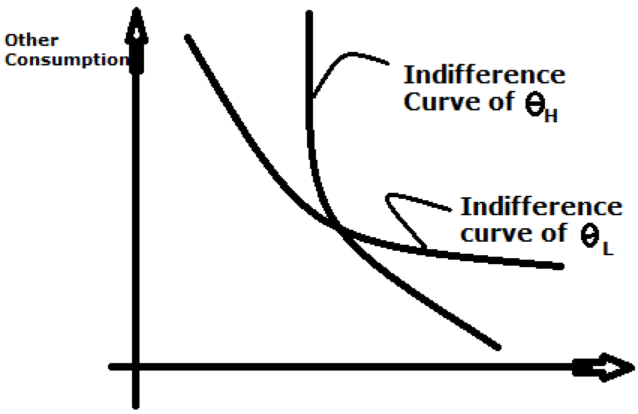

Figure 10.e9 Indifference curves of two types of consumers