

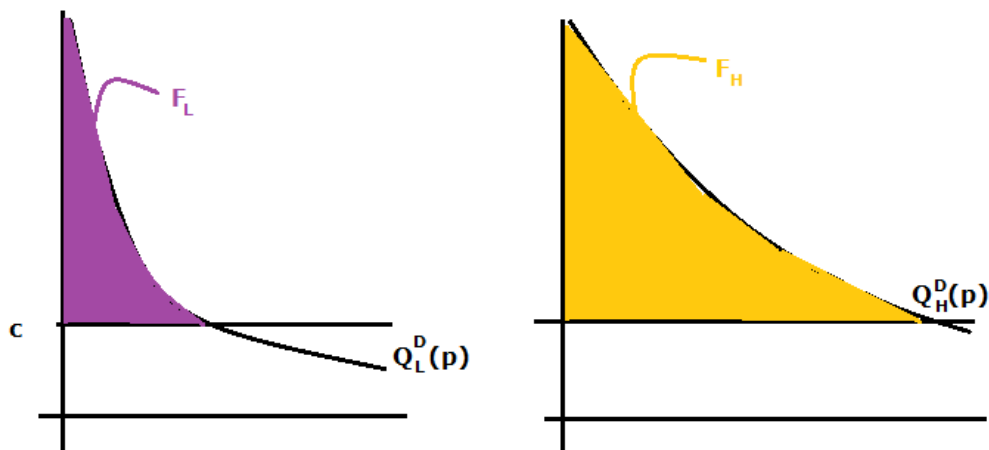
**Introduction to Industrial Organization**  
**Professor: Caixia Shen Fall 2014**  
**Lecture Note 13**

**Price discrimination (ch 10)-2<sup>nd</sup> price discrimination-continue**

How can the monopolist discriminate between  $\theta_H$  and  $\theta_L$ ?

1. If first or third degree price discrimination is possible, problem is easy.

Set  $p_L = p_H = c$  (marginal cost) in order to get both types to consume the efficient amounts. The extract all surplus via the fixed fee.



**Figure 10.e10 Fixed fee in for two types of consumers**

2. If first or third degree price discrimination not possible, firm can engage in 2<sup>nd</sup> price discrimination. Will design two plans  $(F_L, p_L)$  for  $\theta_L$  and  $(F_H, p_H)$  for  $\theta_H$  types. These two plans must satisfy the same for constraints that Starbucks faces when designing its coffee products.

$$1) \theta_H \ln(q_H^*(p_H)) - p_H q_H^*(p_H) - F_H \geq 0$$

$$2) \theta_L \ln(q_L^*(p_L)) - p_L q_L^*(p_L) - F_L \geq 0$$

The two expressions above means that  $\theta_H$  and  $\theta_L$  must get utility  $\geq 0$

$$3) \theta_H \ln(q_H^*(p_H)) - p_H q_H^*(p_H) - F_H \geq \theta_H \ln(q_H^*(p_L)) - p_L q_H^*(p_L) - F_L$$

$$4) \theta_L \ln(q_L^*(p_L)) - p_L q_L^*(p_L) - F_L \geq \theta_L \ln(q_L^*(p_H)) - p_H q_L^*(p_H) - F_L$$

The two expressions above means that  $\theta_H$  must prefer  $(F_H, p_H)$  to  $(F_L, p_L)$  and

$\theta_L$  must prefer  $(F_L, p_L)$  to  $(F_H, p_H)$ .

The firm's profits will be

$$\pi = \max_{F_H, p_H, F_L, p_L} \{F_L + F_H + p_H q_H^*(p_H) + p_L q_L^*(p_L) - c q_H^*(p_H) - c q_L^*(p_L)\}$$

This problem can easily be transformed in such a way that it looks identical to Starbuck's:

$$\text{Let } \tilde{p}_H = F_H + p_H q_H^*(p_H)$$

$$\tilde{p}_L = F_L + p_L q_L^*(p_L)$$

$$\tilde{q}_H = \frac{1}{p_H}$$

$$\tilde{q}_L = \frac{1}{p_L}$$

Firm's problem is now

$$\pi = \max_{\tilde{p}_H, \tilde{p}_L, \tilde{q}_H, \tilde{q}_L} \{\tilde{p}_H + \tilde{p}_L - c\tilde{p}_L - c\tilde{q}_L\}$$

such that

$$\begin{aligned} \theta_H \ln(\tilde{q}_H) - \tilde{p}_H &\geq -\theta_H \ln \theta_H \\ \theta_L \ln(\tilde{q}_L) - \tilde{p}_L &\geq -\theta_L \ln \theta_L \\ \theta_H \ln(\tilde{q}_H) - \tilde{p}_H &\geq \theta_H \ln(\tilde{q}_L) - \tilde{p}_L \\ \theta_L \ln(\tilde{q}_L) - \tilde{p}_L &\geq \theta_L \ln(\tilde{q}_H) - \tilde{p}_H \end{aligned}$$

So the solution will be the same.

- 1) Low types set no utility
- 2) High types indifferent between  $(F_L, p_L)$  and  $(F_H, p_H)$ .
- 3) Can prove that  $p_H = c$  and  $p_L > c$  and  $F_H > F_L$ .

Graphically, we can plot out the two plans.

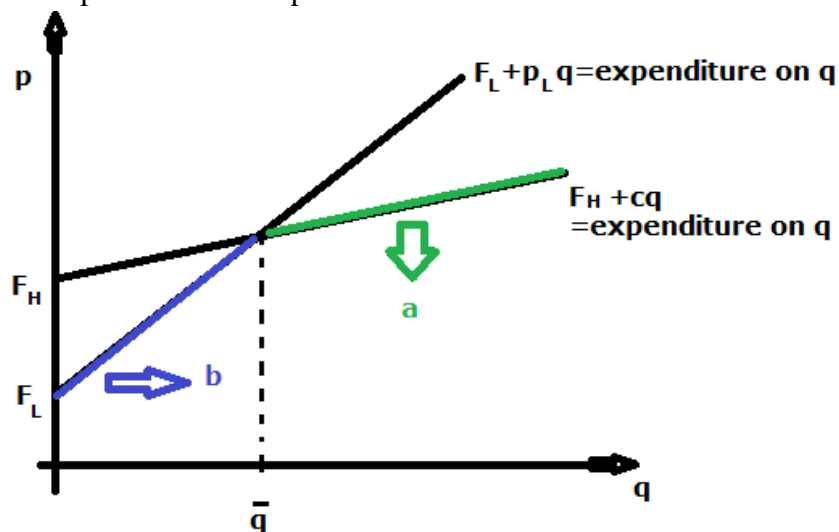


Figure 10.e11 Monopolist 2nd price discrimination

The high types will prefer a bundle along “a”. The low type will prefer a bundle along “b”.

Let other consumption=income-expenditure on q. Then we can plot our results using indifference curves. See Figure 10.e12.

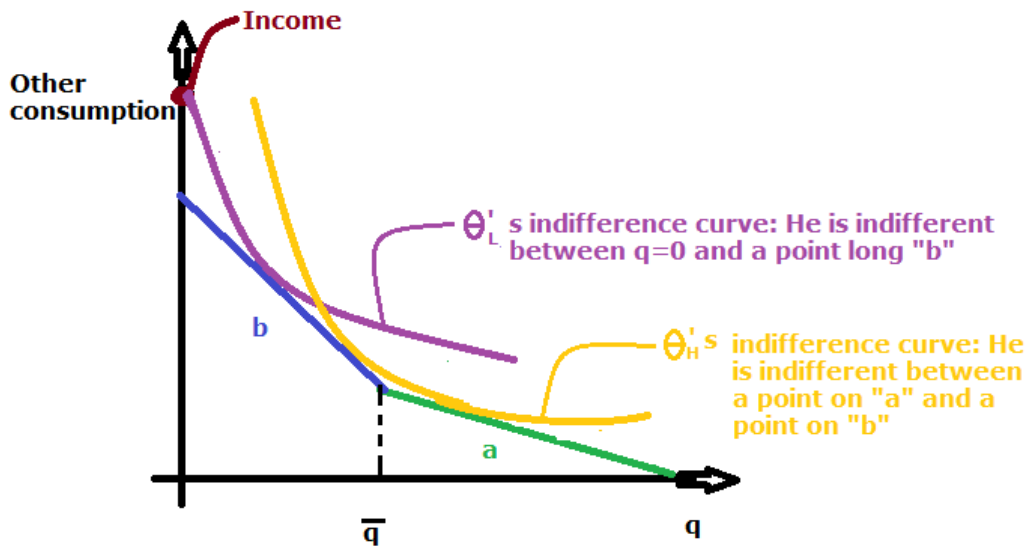


Figure 10.e12: Monopolist 2nd price discrimination-indifference curves

Finally,

- Our model does a good job of duplicating real world two part tariff.
- Zipcar offers two plans.

$$\begin{cases} F_H = 0, p_H = \$6.30 & \text{for } \theta_H \\ F_L = 60, p_L = \$7 & \text{for } \theta_L \end{cases}$$

- If  $\theta_H = 45, \theta_L = 26$ , and  $c = 2.51$  or model products  $F_H = 59.71, F_L = 0.66, p_L = 9.32, p_H = 6.61$

Since we have now finished price discrimination, it shall be interesting to see more on how economists analyze real questions on price discrimination. A short article or a paper (possibly on “theater on Broadway”) will be discussed in the rest of the class.