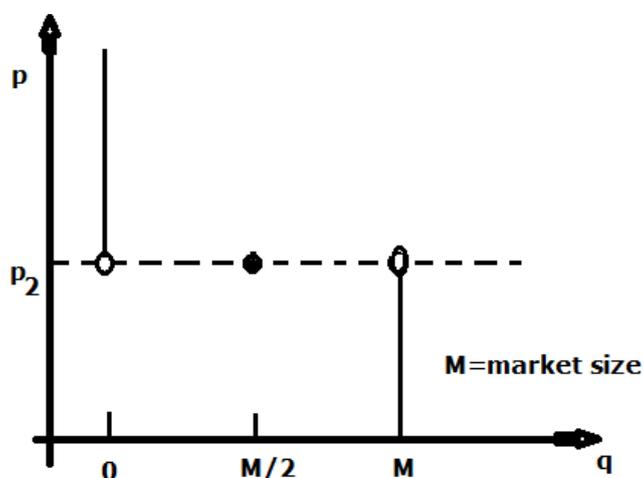


**Introduction to Industrial Organization**  
**Professor: Caixia Shen Fall 2014**  
**Lecture Note 14**

**Product differentiation (Ch. 12)**

This part of the class will look at markets in which firms set prices to maximize profits, but products are differentiated.

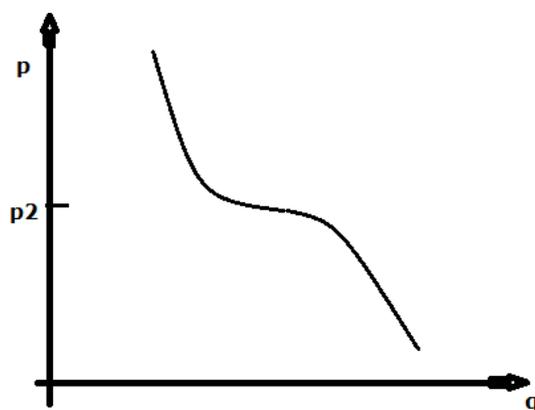
Recall in Bertrand model, we got the equilibrium result that competition drives prices down to marginal costs because the goods are perceived as perfect substitutes. As a result, given  $p_2$ , firm 1's demand curve looks like



**Figure 12.e1 Product differentiation - demand curve 1**

See Figure 12.e1. Firm 1's demand curve is extremely elastic in the neighborhood of  $p_2$ . This makes firm 1's incentive to undercut firm 2's price too strong!

If firm 1's demand curve actually looked like the demand curve drawn below, then lowering price below  $p_2$  will cause some consumers to switch from product 2 to product 1, but not all. This reduces firm 1's incentive to undercut firm 2.



**Figure 12.e2 Product differentiation - demand curve 2**

When firm's demand curve looks like that in Figure 12.e2, we can get the equilibrium result that  $p > c$  in Bertrand games.

Firm 1's demand curve looks like this when product 1 and product 2 are similar (they are substitutes) but not identical. They are differentiated (they are not perfect substitutes) in some way.

Outline of this chapter:

- 1) Solve Bertrand model with differentiated products
- 2) Write down models of consumer choice in order to explain why products can be differentiated (there are several reasons)
- 3) Empirical application

Solving model of Bertrand pricing with different products.

Suppose

$$\begin{aligned} Q_1 &= a - bp_1 + bp_2 \\ Q_2 &= a - bp_2 + p_1 \\ MC &= c \end{aligned}$$

These are downward slopping demand curves and they indicate that good 1 and good 2 are substitutes but not perfect substitutes. If  $p_1 < p_2$ , then  $Q_1 > Q_2$  but  $Q_2$  still  $> 0$ .

Note that  $Q_1 = a + b(p_2 - p_1)$ .

$$\text{If } p_1 = p_2 + 1, \text{ then } Q_1 = a - b.$$

$$\text{If } p_1 = p_2 - 1, \text{ then } q_1 = a + b.$$

So going from "being undercut" to "undercutting" increases demand by only  $2b$  units of output, not entire market.

Intuition: when firm 1 undercuts firm 2, only a small segment of the market finds it worthwhile to switch. Some people don't switch because they actually like product 2 from firm 2.

Solving the model:

- 1) Solve for each firm's BR function.
- 2) Jointly solve the two best response functions.

$$\begin{aligned} \pi_1 &= [a - bp_1 + bp_2](p_1 - c) \\ \frac{d\pi_1}{dp_1} &= -b(p_1 - c) + (a - bp_1 + bp_2) = 0 \end{aligned}$$

$$\rightarrow p_1 = \frac{a + cb + bp_2}{2b} = \frac{a}{2b} + \frac{1}{2}(c + p_2)$$

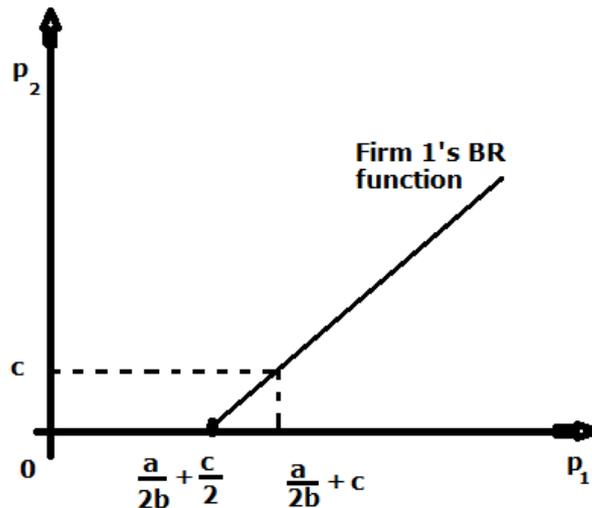


Figure 12.e3 Firm 1's BR function

If  $p_2$  falls by 1, firm 1 will only reduce  $p_1$  by  $1/2$ . Firm 1 is willing to cede a part of the market to firm 2 because he will still make positive sales. Note that sometimes, firm 1 actually find it optimal to set  $p_1 > p_2$ . For example, if  $p_2 = c$ ,  $p_1^* = c + a/2b$ . This never happens in Bertrand.

Similarly:

$$\pi_2 = [a - bp_2 + bp_1](p_1 - c)$$

$$\frac{d\pi_2}{dp_2} = 0 \rightarrow p_2 = \frac{a}{2b} + \frac{1}{2}(c + p_1)$$

Since these best response functions are symmetric, we know that in equilibrium,  $p_1^* = p_2^*$ . Plugging this into a BR function yields

$$p_1^* = \frac{a}{2b} + \frac{1}{2}c + \frac{1}{2}p_1^*$$

$$p_1^* = c + \frac{a}{b}$$

$$p_1^* = p_2^* = c + \frac{a}{b}$$

Notice that

- 1)  $P^* > c$ , so there is a markup over marginal costs and therefore we will not obtain the efficient allocation.

In Figure 12.e4,  $Total\ Surplus = CS_1 + CS_2 + \pi_1 + \pi_2$

Where  $CS_1 + CS_2 = total\ CS$ , and  $total\ profits = \pi_1 + \pi_2$

- 2) As demand shifts outward ( $a$  increases),  $p^*$  increases.
- 3) As consumers become more price sensitive ( $b$  increases),  $p^*$  decreases)

The above two points are just like in Cournot and monopoly models. So when products 1 and

2 are differentiated, we get equilibrium markups that look like Cournot even though firms are competing in prices in Bertrand fashion.

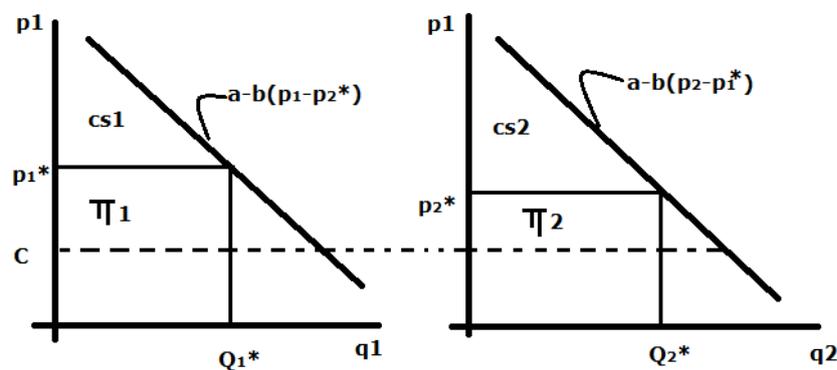


Figure 12.e4: Equilibrium with product differentiation: inefficient allocation ( $p^* > c$ )

What does it mean (at a microeconomic level) for products to be differentiated? If products are differentiated, they are imperfect substitutes. Goods 1 and 2 can be imperfect substitutes for two reasons:

1) Horizontal differentiation

- Some consumers prefer product 1. Other consumers prefer product 2, even when prices are equal. Reasons for these distinct preferences are idiosyncratic.
- Example: Coke vs Pepsi, Windows vs Mac, Credit cards (Visa vs Master cards). In all of these examples, the products are of the same quality, but for some reason consumers usually prefer one or the other.
- Goods might be horizontally differentiated because they are at different locations (and transportation is costly), they have slightly different characteristics (Pepsi is slightly sweeter than Coke, and some people love sweetness), different advertisements, etc.

2) Vertical differentiation

- Everyone agrees that product 1 is better than product 2. But consumers disagree about how important the quality difference is. If  $p_1 = p_2$ , everyone buys product 1. But if  $p_1 > p_2$ , only some consumers find it worthwhile to spend the extra money to purchase product 1.
- Examples: Mercedes vs Toyota, large vs small, Marriott vs Super 8 motel

How do we model horizontal and vertical differentiation? Below, let  $U_{i1}$  and  $U_{i2}$  denote individual  $i$ 's utility from goods 1 and 2. Let  $\delta_1$  and  $\delta_2$  denote the quality level of goods 1 and 2. Let  $p_1$  and  $p_2$  denote prices.

1) Horizontal differentiation ( $\delta_1 = \delta_2 = \delta$ )

$$U_{i1} = \delta - p_1 + \varepsilon_{i1}$$

$$U_{i2} = \delta - p_2 + \varepsilon_{i2}$$

$\varepsilon_{i1}$  and  $\varepsilon_{i2}$  are idiosyncratic shocks to  $i$ 's utility for 1 and 2. They are the source of the horizontal differentiation.

$$\begin{aligned} \text{Purchase 1 if } U_{i1} > U_{i2} &\iff \delta - p_1 + \varepsilon_{i1} > \delta - p_2 + \varepsilon_{i2} \\ &\iff p_2 - p_1 > \varepsilon_{i2} - \varepsilon_{i1} \end{aligned}$$

If  $p_1 = p_2$ , buy good 1 if  $\varepsilon_{i1} > \varepsilon_{i2}$ .

If  $p_1 = p_2$ , buy good 2 if  $\varepsilon_{i1} < \varepsilon_{i2}$ .

$\varepsilon_{i1}$  and  $\varepsilon_{i2}$  represent idiosyncratic shocks to I's utility. If goods 1 and 2 are

- Restaurants:  $\varepsilon_{i1}$  and  $\varepsilon_{i2}$  might = -1 × distance from I's home to the restaurants
- Credit cards:  $\varepsilon_{i1}$  and  $\varepsilon_{i2}$  might equal utility from cash back and divided miles.
- Windows vs Mac:  $\varepsilon_{i1}$  and  $\varepsilon_{i2}$  might equal importance individual i places on business application and style.

Main point: even if  $\delta_1 = \delta_2$  and  $p_1 = p_2$ , some consumers will strictly prefer good 1, other will strictly prefer 2.

## 2) Vertical differentiation

- Assume that product 1 higher quality than 2 ( $\delta_1 > \delta_2$ )
- But consumer vary in how much they value quality

Let  $U = \theta\delta - p$  so that  $\frac{dU}{d\delta} = \theta$ . Consumer i willing to pay  $\theta_i$  dollars for extra unit of quality.

- Thus,

$$U_{i1} = \theta_i \delta_1 - p_1$$

$$U_{i2} = \theta_i \delta_2 - p_2$$

If  $p_1 = p_2$ , then everyone will purchase good 1 since  $\delta_1 > \delta_2$ .

$$\begin{aligned} U_{i1} > U_{i2} &\iff \theta_i \delta_1 - p_1 > \theta_i \delta_2 - p_2 \\ &\iff \delta_1 > \delta_2 \end{aligned}$$

If  $p_1 > p_2$ , then I will purchase good 1 if he cares a lot about quality

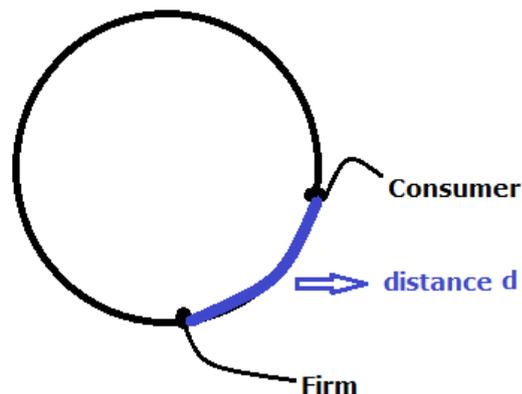
$$\begin{aligned} U_{i1} > U_{i2} &\iff \theta_i \delta_1 - p_1 > \theta_i \delta_2 - p_2 \\ &\iff \theta_i \delta_1 - \theta_i \delta_2 > p_1 - p_2 \\ &\iff \theta_i > \frac{p_1 - p_2}{\delta_1 - \delta_2} \end{aligned}$$

Usually we think that income is an important determinant of  $\theta$ . High income people are willing to pay more for quality than low income people. But the determinants of  $\theta$  depends upon that differentiates the products. For example if good 1=fast computer and good 2=slow computer, then  $\theta_i = i$ 's need for computing power. Or if 2=less generous health insurance and 1=generous health insurance,  $\theta_i = i$ 's health.

We have introduced a way to think about product differentiation. Is all of the product variety we observe in the real world a good thing? Consumers benefit from variety. But introducing products is costly. Do competitive firms introduce too much variety?

Consider Salop circle model:

Consumer and firms are located along a unit circle.



**Figure 12.e5 Salop circle model**

A consumer would like for firm to be located at his location. But since one is not, consumer has to travel a distance  $d$  to get firm. This entails travel costs  $ct$ .

Setup:

- Consumers purchase good from firm offering most utility. If distance equals  $d$ , consumer get utility  $U-p-cd$ . All consumer buy from one firm.
- If there are  $N$  firms, they are located  $1/N$  units apart
- $N$  firms set prices in Bertrand fashion
- Marginal costs=0, fixed costs of entry=0

We will solve for the efficient amount of product variety and the amount that emerges in equilibrium.

Suppose there are  $N$  firms. What is total surplus?

If a consumer is located “ $d$ ” units away from a firm, and he makes the purchase, the surplus from this purchase= $CS+PS=U-cd-p+p=U-cd$ . Where  $U-cd-p$  is CS, and  $p$  is PS.

Which consumers purchase from which firms? Assume prices are all equal. Consider two firms:

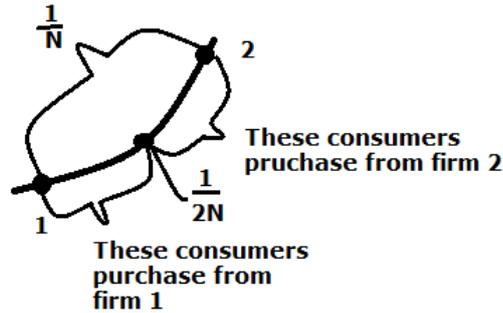


Figure 12.e6 Salop circle model-distance

So the presence of firm 1 generates surplus equal to :

$$\begin{aligned} \text{Surplus from 1 firm} &= 2 \int_0^{1/2N} (U - ct) dt - F = 2[Ux - \frac{1}{2}cx^2]_0^{1/2N} - F \\ &= \frac{U}{N} - \frac{c}{4N^2} - F \end{aligned}$$

N firms combined generate surplus.

$$TS \text{ from } N \text{ firms} = N \left( \frac{U}{N} - \frac{c}{4N^2} - F \right) = U - \frac{c}{4N} - FN$$

Surplus is maximized at

$$\frac{dTS}{dN} = \frac{c}{4N^2} - F = 0$$

$$\rightarrow N^* = \sqrt{\frac{c}{4F}}$$

The efficient # of firms  $= \sqrt{\frac{c}{4F}}$ . As transportation costs increases,  $N^*$  increase since consumer benefit more from variety. As  $F$  increases,  $N^*$  decreases since the costs of introducing more variety increases.

What  $N$  emerges in a competitive equilibrium?

If firms are located  $1/N$  apart, what prices will they set and what will profits be? This will determine how many products enter market.

Suppose firm 2 competes against firms 1 and 2, who both charge a price  $p'$ . What is the optimal price for firm 2?

See Figure 12.e7.

If a consumer is located distance  $t$  from 2, he is  $1/N - t$  from firm 3. To calculate 2's market share given  $p$  and  $p'$ , look for indifferent consumer

$$U - p - ct = U - p' - c\left(\frac{1}{N} - t\right)$$

$$t = \frac{p' - p}{2c} + \frac{1}{2N}$$

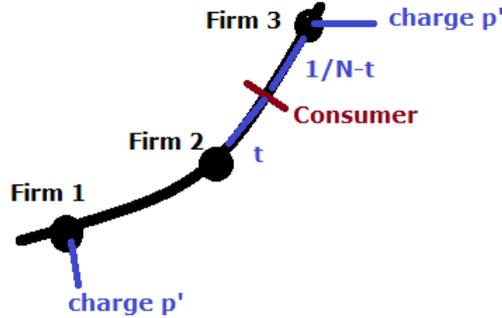


Figure 12.e7: Salop circle model-3 firms

Firm 2, if he charges a price  $p$ , will sell  $2\left[\frac{p' - p}{2c} + \frac{1}{2N}\right]$  units of output. As price  $p$  increases, he sells less. As competitor's prices increases, he sells more.

So:

$$\pi_{Firm\ 2} = 2p \left[ \frac{p' - p}{2c} + \frac{1}{2N} \right] - F$$

Firm 2 will choose  $p$  to maximize profits.

$$\frac{d\pi}{dp} = 2 \left( \frac{p' - p}{2c} + \frac{1}{2N} \right) + 2p \left( -\frac{1}{2c} \right) = 0$$

$$\frac{p'}{c} - \frac{p}{c} + \frac{1}{N} - \frac{p}{c} = 0$$

$$p = \frac{1}{2} \left( \frac{c}{N} + p' \right)$$

As  $c$  increases, price  $p$  increases. As  $N$  increases, price  $p$  decreases. As  $p'$  increases, price  $p$  increases.

In symmetric equilibrium  $p = p'$  (since all firms identical) and  $p = \frac{c}{N}$ .

$$\text{If } p = \frac{c}{N}, \text{ then } \pi = 2 \int_0^{\frac{1}{2N}} p - F = \left( \frac{2c}{N} \right) \left( \frac{1}{2N} \right) - F = \frac{c}{N^2} - F$$

Firms will keep entering the market as long as  $\pi > 0$ . so in equilibrium,  $N^{equal}$  firms will enter so that  $\frac{c}{N^{equal}^2} - F = 0 \iff N^{equal} = \sqrt{\frac{c}{F}}$

In the competitive equilibrium,  $\sqrt{\frac{c}{F}}$  types of products exit. The social planner prefers  $\frac{1}{2} \sqrt{\frac{c}{F}}$  types of products. Competitor yields too much product differentiation.

Intuition:

When a firm enters, he imposes costs and benefits on society. Costs are fixed costs  $F$ , and reduced profits for other firms (business stealing). Benefits are the increased CS through product variety. Firm internalizes  $F$  and benefits of product variety (through price).