Introduction to Industrial Organization
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Lecture Note 5

Games and Strategy (Ch. 4)

Outline:
Modeling by means of games
Normal form games
Dominant strategies; dominated strategies, Iterated elimination of dominated strategies
Nash equilibrium
Preview: Cournot model of duopoly
Preview: Bertrand model of duopoly
Sequential games: backward induction and perfect, commitment
Preview: Stackelberg model of duopoly
Repeated games

1. Modeling by means of games
    - ➢ Modeling situations where the payoff for one agent depends on its own actions as well as the actions of the other agents
    - ➢ In these situations what matters is strategic behavior:
        - ✓ Optimal choice for an agent (optimal action or strategy) depends on what it conjectures other players will choose
        - ✓ Other players act similarly, so may need to conjecture what the other players conjecture and so forth
        - ✓ If strategic interaction over time, should take into account future (impacts and actions)
2. Normal form games
    - ➢ A game consists of
        - ✓ A set of player
        - ✓ A set of rules
        - ✓ A set of rules
        - ✓ A set of payoffs: the utility each player gets as result of each possible combination of strategies
    - ➢ Representing this in a matrix, with cells corresponding to the combination of strategic choices: normal form
    - ➢ One-short game: players choose their strategies simultaneously (do not observe other's choice).

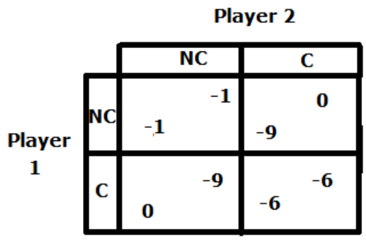


Figure 4.1 The "prisoner's dilemma" game

I illustrate the normal-form representation with a classic example-*the prisons' dilemma* (figure 4.1). Two suspects are arrested and charged with a crime. The police lack sufficient evidence to convict the suspects, unless at least one confesses. The police hold the suspects in separate cells and explain the consequences that will follow from the actions they could take. If neither confesses then both will be convicted of a minor offense and sentenced to one month in jail. If both confess then both will be sentenced to jail for six months. Finally, if one confesses but the other does not, then the confessor will be released immediately but the other will be sentenced to nine months in jail.

The prisons' problem is illustrated in Figure 4.1. In this game, each player has two strategies available: confess (C) and not confess (NC). The payoff to a particular pair of strategies is chosen are given in the appropriate cell of the matrix.

The *normal-form representation* of a game specifies: 1) the players in the game; 2) the strategies available to each player; 3) the payoff received by each player for each combination of strategies that could be chosen by the players.

In this game, players are player 1 and player 2. The strategies available to each player are {C, NC}. The payoff received by each player is represented in the matrix.

3. Dominant strategies; dominated strategies, Iterated elimination of dominated strategies

Strategy s_i is *strictly dominant strategy* if for each feasible combination of the other players' strategies, i's payoff from playing s_i is strictly *larger than any other strategy*. If the players are rational, and if a player has a dominant strategy, we should expect the player to choose the dominant strategy.

More general, we like to define *dominated strategy*. Strategy s_i^1 is strictly dominated by strategy s_i^2 if for each feasible combination of the other players' strategies, i's payoff from playing s_i^1 is strictly less than i's payoff from playing s_i^2. The idea is that if a given player has a dominated strategy and that player is rational, then we should expect the player not to choose such a strategy.

Iterated elimination of dominated strategies

Since a rational player will not play a dominated strategy, we can a method "iterated elimination of dominated strategies" to eliminate these strategy. Something more about a game can be said if we eliminate dominated strategies, if any.

In figure 4.2, for example, player 2 should expect player 1 not to choose M. Given that player 1 does not choose M, player 2 finds strategy C to be dominated either by L or R. We can now take this process further. If both players are rational and they know the fact that both are rational, player 1 should find T to be a dominated strategy. Finally, we conclude that L is a dominated strategy for player 2. This leaves us with pair of strategies (B,R).

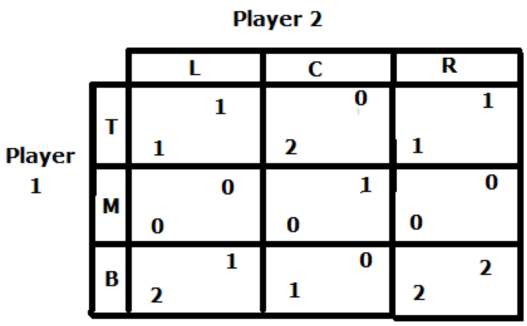
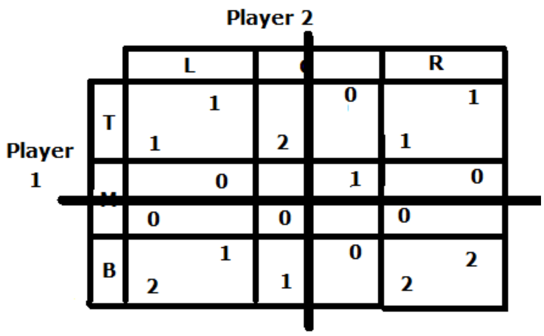


Figure 4.2 Iterated elimination of dominated strategies



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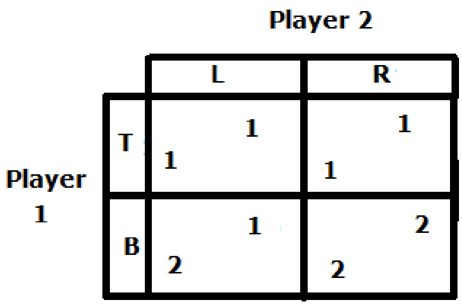

Figure 4. 2 Iterated elimination of dominated strategies

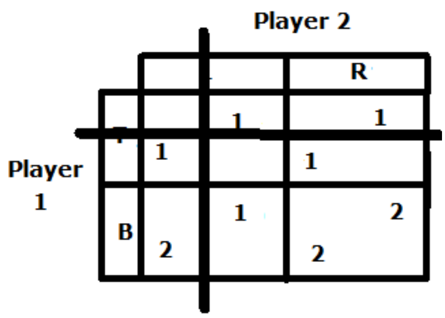

Figure 4. 2 Iterated elimination of dominated strategies

By using the iterated elimination of dominated strategy, the final solution to this game is (B, R) with payoff (2, 2).

The very important condition here is that all the players are rational and they all believe each other are rational. The beliefs are important.

Note that it is rare that in a game, we can successfully eliminate dominated strategies in each step and finally get one solution. Most games may not have dominated strategy since it requires a strong condition. So what are we going to do if we can not use iterated elimination of dominated strategy?

Now we are going to talk about Nash Equilibrium.

4. Nash equilibrium

A pair of strategies constitutes a Nash Equilibrium if no player can change its strategy in a way that improves its payoff, holding constant the other players equilibrium strategies.

For example the prisons' dilemma game above. Let's find out the Nash equilibrium. The way to find a NE or multiple NEs is to hold other players strategies constant, then to find out a best response for one player. Keep doing this for all players, find out a stable situation where no one wants to deviation.

		Player 2	
		NC	C
Player 1	NC	-1, -1	-9, 0
	C	0, -9	-6, -6

Figure 4.1 The "prisoner's dilemma" game

In this game, let's solve NE by steps.

1) Let's suppose player 2 playing NC. Then Player 1's best response is to play C since 0 is better than -1 given player 2 plays NC.
2) Let's suppose now player 2 playing C. then player 1's best response is to play C since -6 is better than -9 given player 2 players C.
3) Now we know that player 1 will always play C no matter which strategy player 2 plays.
4) Given player 1 plays C, player's best response is C since -6 is better than -9.

Thus, (C, C) is a NE since in this combination of strategies, no one wants to deviate.

Multiple Nash Equilibrium

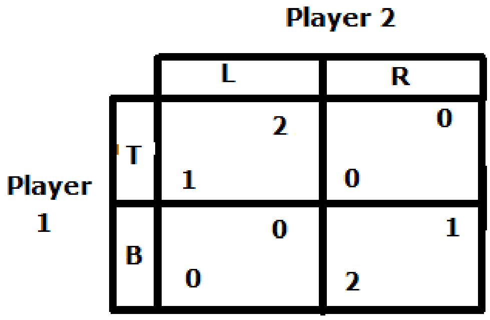

Figure 4.5 Multiple Nash Equilibrium

Note that it is not necessary there is only one Nash Equilibrium. In figure 4.5, this game has two Nash Equilibria.

More advanced:
Nash Theorem: In any finite game (finite number of players, finite number of strategies), there exists a Nash Equilibrium (possibly involving a mixed strategy NE).

5. Cournot model of duopoly

Now we are going to talk about the IO application of game theory. In this section we will use the Cournot model to illustrate the computations involved in solving for game's Nash equilibrium.

➢ Nash equilibrium in Cournot model of duopoly

q1 and q2 denote the quantities produced by firms 1 and 2, respectively. Let p(Q)=a-Q be the market-clearing price when the aggregate quantity on the market is Q= q1+q2. Assume that the total cost to firm is C(q1)=cq1 and C(q2)=cq2.

In order to find the Nash equilibrium of the Cournot model, we first translate the problem into a normal-form game: 1) the players: two firms; 2) the strategies available to each player: different quantities each firm produce; 3) the payoff received by each player: profit.

Thus, the profit-maximizing objective for firm i is

$$\max_{q_i} \quad \pi_i(q_i, q_j) = q_i\big(p(q_i + q_j) - c\big) = q_i\big(a - (q_i + q_j) - c\big)$$

$$\text{FOC: } q_i = \frac{1}{2}(a - q_j^* - c)$$

Thus if the quantity pair (q_1*, q_2*) is to be a Nash equilibrium, the firms' quantity choices must satisfy

$$q_1^* = \frac{1}{2}(a - q_2^* - c)$$
and
$$q_2^* = \frac{1}{2}(a - q_1^* - c)$$

Solving this pair of equations yields $q_1^* = q_2^* = \frac{a-c}{3}$.

Note this quantity is below the monopoly quantity is $q_m^* = \frac{a-c}{2}$. Why not each firm produce half of the monopoly quantity, i.e. $\frac{a-c}{4}$? In this case, both firm want to produce more than half of the monopoly quantity since the price p_m^* at monopoly market-clearing quantity q_m^* is high and each firm wants to make profit from p_m^*. Thus each produce more than $\frac{a-c}{4}$, thus market − clearing price now is lower than the monopoly price.

- Best response function

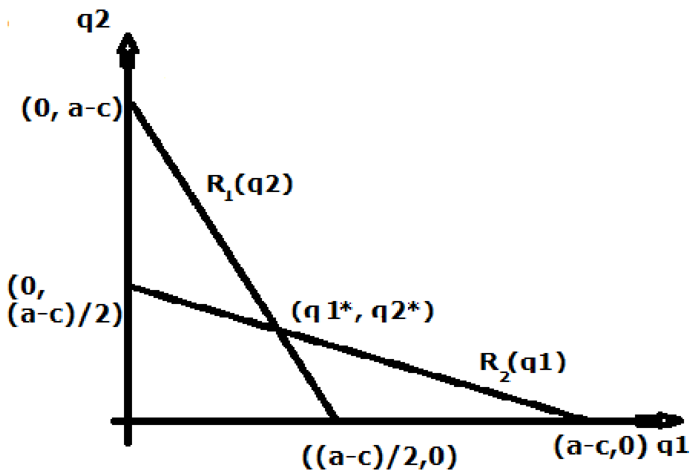

Figure 4.e1 Best reponse function in Cournot model

Rather than solve the NE algebraically, one can draw a graph of best response functions. For any quantity firm one produce, the firm 2's best response function is

$$R_2(q_1) = \frac{1}{2}(a - q_1 - c)$$

And same for firm 1's best response function is

$$R_1(q_2) = \frac{1}{2}(a - q_2 - c)$$

Figure 4.e2 shows these two best-response functions intersect only once, at the equilibrium pair (q_1^*, q_2^*).

6. Bertrand model of duopoly

We next consider a model of how two duopolists might interact when firms actually chooses prices rather than quantity.

If firm 1 and firm 2 choose prices p1 and p2, respectively, the quantity that consumers demand from firm i is

$$q_i(p_i, p_j) = a - p_i + bp_j$$

We again assume that the payoff function for each firm is just its profit. The profit to firm I when it chooses the price p_i and its rival chooses the price p_j is

$$\pi_i(p_i, p_j) = q_i(p_i, p_j)[p_i - c] = a - p_i + bp_j][p_i - c]$$

Thus, the price pair (p_1*, p_2*) is a nash equilibrium if, for each firm i p_i* solves

$$\max \quad a - p_i + bp_j][p_i - c]$$

The solution to firm i's optimization problem is

$$p_i = \frac{1}{2}(a + p_j^* + c)$$

Therefore, if the price pair (q_1^*, q_2^*) to be Nash equilibrium , the firms' price choices must satisfy

$$p_1^* = \frac{1}{2}(a + bp_2^* + c)$$

and

$$p_2^* = \frac{1}{2}(a + bp_1^* + c)$$

Solving this pair of equations yields

$$p_1^* = p_2^* = \frac{a + c}{2 - b}$$