Introduction to Industrial Organization
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Lecture Note 6

Games and Strategy (ch.4)-continue

Outline:
Modeling by means of games
Normal form games
Dominant strategies; dominated strategies, Iterated elimination of dominated strategies
Nash equilibrium
Preview: Cournot model of duopoly
Preview: Bertrand model of duopoly
Sequential games: subgame-perfect equilibrium and commitment
Preview: Stackelberg model of duopoly
Repeated games

7. Sequential games: backward induction and perfect, commitment

Previously we learned simultaneous-move games, now we are going to talk about dynamic games.

Consider an example of an industry that is currently monopolized. A second firm must decide whether or not to enter the industry. Given this situation, the incumbent firm must decide whether to price aggressively or not. The incumbent's decision is taken as a function of the entrant's decision. This means entrant move first, and in incumbent moves second.

Unlike in static game using normal form representation, we will use a game tree (extensive-form representation) to illustrate a dynamic game. An example is given in figure 4.6.

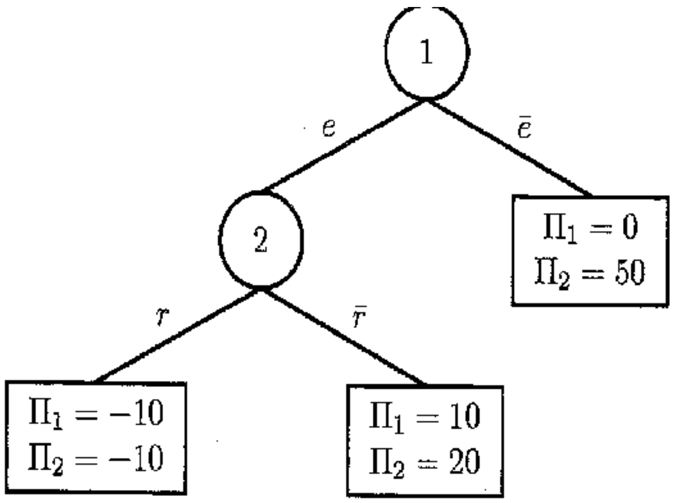

Figure 4.6 Extensive-form representation: The sequential-entry game

In figure 4.6, a circle denotes a decision node. The game starts with decision n node 1. At this node, player 1 (the entrant) makes a choice between $e \; and \; \bar{e}$, which means enter or not enter. If the entrant does not enter, then the game ends with payoffs $\pi_1 = 0$ (the entrant's payoff) and $\pi_2 = 50$ (the incumbent's payoff). If player choose enter, then we move on to decision node 2. This node corresponds to player 2 making a choice between $r \; and \; \bar{r}$, which means retaliate entry or not retaliate entry.

There are two Nash equilibria in this game: (e , $\bar{r}$) and ($\bar{e}, r$). Let's now check that both are NEs.

First, for (e , $\bar{r}$) to be an NE, no players has no incentive to change its strategy given what the other player does. First, if player 1 chooses e, then player 2's best choice is to choose $\bar{r}$, since it gets 20 otherwise it would get -10. Likewise, given that player 2 chooses $\bar{r}$, player 1' best response is to choose e, it gets 10 otherwise it would get 0.

Second, for ($\bar{e}, r$) to be an NE, no player wants to deviate its strategy given the other player's equilibrium strategy. Given that player 2 chooses r, player 1 is better off by choosing $\bar{e}$: this yields player a payoff of 0, where e would yield -10. As for player 2, given that player 1 plays $\bar{e}$, its payoff is 50, regardless of which strategy it chooses. It follows that r iis an optimal strategy, although it is not the only one.

Credible threat or not?

Although two solutions are indeed two Nash equilibria, the second equilibrium does not make much sense. Player 1 is not entering because of the "threat" that player 2 will choose to retaliate. But is this threat credible? If player 1 were to enter, would player 2 decide to retaliate? Clearly, the answer is NO. by retaliating, player 2 gets -10, compared with 20 from no retaliation. We conclude that $(\bar{e}, r)$ is not a reasonable prediction of what one might expect to be played, although it is an NE.

Backward induction

Then we must think of a finer solution concept to get rid of this type of unreasonable NEs. One way to solve this problem is to solve the game backward, that is, to apply the principle of backward induction.

We first consider node 2, and conclude that optimal decision is $\bar{r}$. Then, we solve for the decision in node 1 given the decision previously found for node 2. Given that player 2 will choose $\bar{r}$, it is now clear that the optimal decision at node 1 is e. we thus select the first Nash equilibrium as the only one that is intuitively reasonable.

Subgame perfect equilibrium

Now think of that player chooses e at decision node 1. We are not led to player 2 decision node but rather to an entire new game, a simultaneous-move game as in figure 4.5. Because this game is part of the game, we call it a subgame of the larger one. In this setting, solving the game backward would amount to first solving for the Nash equilibrium of the subgame, and then given the solution for the subgame, solving for the entire game. Equilibria that are derived in this way are called subgame-perfect equilibria.

In the game of figure 4.6, the equilibrium $(\bar{e}, r)$ was dismissed on the basis that ir requires player 2 to make the incredible commitment of playing r in case player 1 chooses e. Such threat is not credible because, given that player 1 has chosen e, player 2's best choice is $\bar{r}$. But suppose that player 2 writes an enforceable and non renegotiable contract whereby, if player 1 chooses e, player 2 chooses r. The contract is such that player 2 were not to choose r and chose $\bar{r}$ instead, player 2 would incur a penalty of 40, lowering its total payoff to -20.

The situation is illustrated in figure 4.7.

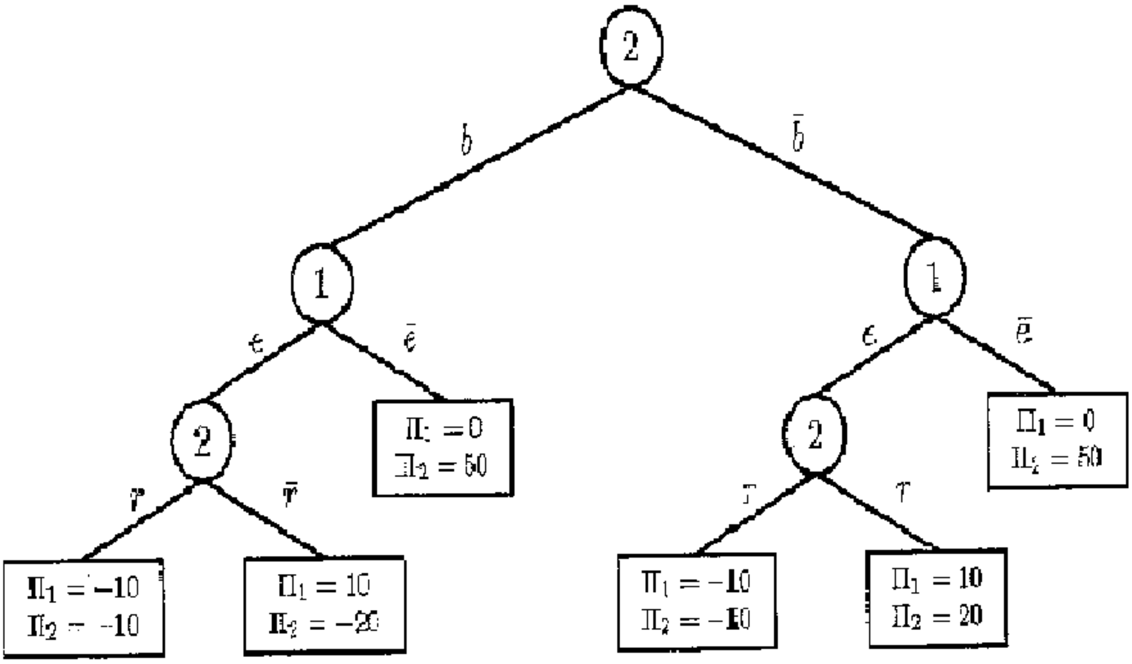

Figure 4.7 The value of commitment

The first decision now belongs to player 2, who must choose between writing the bond described above (strategy b) and not doing anything (strategy b bar). If player 2 chooses $\bar{b}$, then the game in figure 4.6 is played. If instead player 2 chooses b, then a different game is played, one that takes into account the implications of signing the bond.

Let us solve the left-had subgame backward,. When it comes to player 2 to choose between r and $\bar{r}$, the optimal choice is $\bar{r}$. in fact, this gives player 2 a payoff of -10, whereas the alternative would yield -10 (player 2 would have to pay for breaking the bond). Given that player 2 chooses r, player 1 finds it optimal to choose $\bar{e}$. It is better to receive a payoff of zero than to receive -10, the outcome of e followed by r. In summary, the subgame on the left-hand side gives player 2 an equilibrium payoff of 50, the result of combination of is $\bar{e}$ and r.

We can finally move back to one more stage and look at player 2's optimal choice between b and $\bar{b}$. From what saw above, player 2's optimal choice is to choose b and eventually receive a payoff of 50. The alternative, $\bar{b}$, eventually leads to a payoff of 20 only.

This example illustrates two important points: first it shows that *a credible commitment may have significant strategic value.* By signing a bond that imposes a large penalty when playing $\bar{r}$, player 2 credibly commits to playing r when the time comes to choose between r and $\bar{r}$. In doing so, player 2 induces player 1 to choose $\bar{e}$, which in turn works for player 2's benefit. Specifically, introducing this credible commitment raises player 2's payoff from 20 to 50. The value of commitment is 30 in this example.

Second, if we believe that player 2 is credibly committed to choosing r, then we should model this by changing player 2's payoffs or by changing the order of moves. This can be done as in figure 4.8. (page 58)

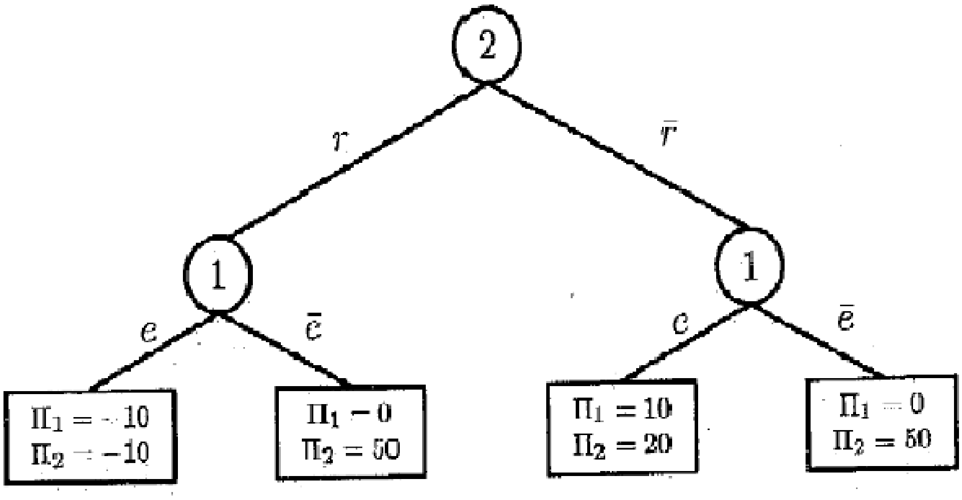

Figure 4.8 Modeling player 2's capacity to precommit

In this game, we model player 2 as choosing r or r bar before player 1 chooses its strategy. The actual choice of r or r bar may occur in time after player 1 chooses e or e bar. However, if player 2 precommits to playing r, we can model that by assuming player 2 moves first. In fact, by solving the game in figure 4.8 backward, we get the same solution as in figure 4.7, namely the second Nash equilibrium of the game initially considered.

8. Stackelberg model of duopoly

Stackelberg (1934) proposed a dynamic model of duopoly in which a dominant (or leader) firm moves first and a subordinate (a follower) firm moves second. At some points in the history of the U.S. automobile industry, for example, General Motors has seemed to play such a leadership role. It is straightforward to extend what follows to allow for more than one following firm, such as Ford, Chrysler, and so on. Following Stackelberg, we will develop the model under the assumption that the firms choose quantities, as in the Cournot model.

The timing of the game is as follows: 1) firm 1 chooses a quantity $q_1>0$; 2) firm 2 observes q_1 and then chooses a quantity $q_2>=0$; 3) the payoff to firm i is given by the profit function

$$\pi_i(q_i, q_j) = q_i[p(Q) - c]$$

Where $p(Q)=a-Q$ is the market-clearing price when the aggregate quantity on the market is $Q=q_1+q_2$, and c is the constant marginal cost of production, fixed cost is zero.

To solve for the backward-induction outcome of this game, we first compute firm 2's reaction to an arbitrary quantity by firm 1. $R_2(q_1)$ solves

$$\max \pi_2(q_1, q_2) = \max q_2[a - q_1 - q_2 - c]$$

Which yields

$$R_2(q_1) = \frac{a - q_q - c}{2}$$

Provided $q_1<a-c$. Since firm 1 can solve firm 2's problem as well as firm 2 can solve it, firm 2 should anticipate that the quantity choice q_1 will be met with reaction $R_2(q_1)$. Thus, firm 1's problem in the first stage of the game amounts to

$$\max \pi_1(q_1, R_2(q_1)) = \max q_1[a - q_1 - R_2(q_1) - c] = \max q_1 \frac{a - q_1 - c}{2},$$

Which yields

$$q_1^* = \frac{a - c}{2} \;\; and \;\; R_2(q_1^*) = \frac{a - c}{4}$$

As the backwards-induction outcome of the Stackelberg duopoly game. Recall we learned last class that in the Nash equilibrium of the Cournot game each firm produces (a-c)/3. Thus, aggregate quantity in the backwards-induction outcome of the Stackelberg game, 3(a-c)/4, is greater than aggregate quantity in the Nash equilibrium of the Cournot game, 2(a-c)/3, so the market-clearing price is lower in the Stackelberg game. In the Stackelberg game, however, firm 1 could have chosen its Cournot quantity, (a-c)/3, in which case firm 2 would have responded with its Cournot quantity. Thus, in the Stackelberg, firm 1 could have achieved its Cournot profit level but chose to do otherwise, so firm 1's profit in the Stackelberg game must exceed its profit in the Cournot game. But the market-clearing price is lower in the Stackelberg game, so aggregate profits are lower, so the fact that firm 1 is better off implies that firm 2 is worse off in the Stackelberg than in the Cournot game.

The observation that firm 2 does worse in the Stackelberg than in the Cournot game illustrates an important different between single-and multi-person decision problems. In single-person decision theory, having more information can never make the decision maker worse off. In game theory, however, having more information can make a player worse off.

In the Stackelberg game, the information in question is firm 1's quantity: firm 2 know q1, and firm 1 knows that firm 2 knows q1. To see the effect this information has, consider the modified sequential-move game in which firm 1 chooses q1, after which firm 1 chooses q2 but does so without observing q1. If firm 2 believes that firm 1 has chosen its Stackelberg quantity q1*=(a-c)/2, then firm 2's best response is again $R_2(q_1^*)$=(a-c)/4. But if firm 1 anticipates that firm 2 will hold this belief and so choose this quantity, the firm 1 prefers to choose its best response to (a-c)/4, rather than its Stackelberg quantity (a-c)/2. Thus, firm 2 should not believe that firm 1 has chosen its Stackelberg quantity. Rather, the unique Nash equilibrium of this modified sequential-move game is for both firms to choose the quantity (a-c)/3---precisely the Nash equilibrium of the Cournot game, where the firms move simultaneously. Thus, having firm 1 know that firm 2 knows q1 hurts firm 2.

9. Repeated games

A useful way to model the situation whereby players react to each other's strategic moves is to consider a repeated game. Consider a simultaneous-choice game like the one in figure 4.1. Because in this game each player chooses one action only once, we refer to it as a one-shot game. A repeated game is defined by a one-shot game which is repeated a number of times.

In one-shot game, the strategy is easy to define. Like figure 4.10.

Player 1 \ Player 2	L	C	R
T	5 5	6 3	0 0
M	3 6	4 4	0 0
B	0 0	0 0	1 1

Figure 4.10 One-shot game

In this game, each player has three actions/strategies to choose from: T, M, B for player 1; and L, C, R for each player 2.

Now suppose this one-shot game is repeated twice. In each period, player 1 still has three actions to choose from. However, the set of possible strategies for player 1 is now much more complex. A strategy for player 1 has to indicate what to choose in period 1 and what to choose in period 2 as a function of the actions that were taken in period 1. There are 9 possible outcomes in the first period, three possible actions in the second period, and three possible actions in the first period, player 1 has 3 times 3 to the power of 9, or 50,049 possible strategies.

In the one-shot game, the Nash equilibria are (M,C) and (B,R). One first observation is that the repeated play of the equilibrium strategies of the one-shot game forms an equilibrium of the repeated game. For example, (M,C) in both periods is an equilibrium. This implicit strategies that lead to such equilibrium of the repeated game are, or player 1 choose M in period1 and choose M in period 2 regardless what happened in period 1, and likewise for player 2. That is players choose history-independent strategies.

Whether there are equlibria of the repeated game that do not correspond to equilibria of the one-shot game. Consider the following strategy for player 1: play T on period 1. In period 2, play M if period 1 actions were (T,L); otherwise, play B. As for player 2, take the following strategy: play L in period 1. In period 2, play C if period 1 actions were (T,L); otherwise play R.

And check this is an equilibrium for this repeated game.