Introduction to Industrial Organization
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Lecture Note 7

Oligopoly (ch. 7)

Oligopoly: A small number of firms, each firm takes into account rival's current actions and likely responses to its own action.

- A frequent situation in many industries
- That we will mostly discuss with examples of duopoly

Examples of oligopoly: any industry where only a few firms make virtually all of the sales.

- The top for cereal producers sell 90% of all cereal
- Automobile manufactures. Only a few large producers. Like, Honda must take into account Toyota's decisions when making its own choices.

The key elements which are same across different models of:

- Consumer are price takers
- Homogenous products
- Number of firms is fixed at a small number
- Each firm sets its price or output level.

The key elements vary across different models of oligopoly:

- Whether firms set output (Cournot) or prices (Bertrand)
- One period or multiple periods
- Whether one firm has an advantage over other (Stackelberg)

Questions we will ask:

- What determines equilibrium allocation?
- How do allocations and surplus levels vary with assumptions?
- Are our models consistent with the empirical evidence?

First Model: Cournot Model with 2 firms

- Assume firm 1 produce q1 units of output, firm 2 produces q2
- So total output Q=q1+q2
- What price do firms receive? Depends on aggregate demand. Let p(Q)=a-bQ
- Firms maximize expected profits with total costs=cq
- Notive how firms output choices affect each other. If firm 1 increases q1, price p reduces. This gives firm 2 incentive to produce less output
- Firms choose output levels based on what they expect their competition to do.

Profits for firm 1:

$$\pi_1 = p(q_1 + q_2)q_1 - TC(q_1)$$

Note if firm 1 expects firm 2 to produce q_2 units of output, then firm 1 knows it will receive a price $p(q_1+q_2)$ given its own choice of output.

Profit maximization implies MR=MC as usual, but now MR depends on the other firm's actions. How do we solve the model? Want to look for a (q1*, q2*) such that

- Given that firm 1 expects firm 2 to produce q_2*, firm's MR=MC at q_1*.
- Give that firm 2 expects firm 1 to produce q_1*, firm 2's MR=MC at q_2*.

Solving the model. First look at firm 1:

$$\pi_1 = p(q_1 + q_2)q_1 - TC(q_1)$$
$$= (a - bq_1 - bq_2)q_1 - cq_1$$
$$\frac{d\pi_1}{dq_1} = a - 2bq_1 - bq_2 - c = 0$$
$$\rightarrow \qquad q_1^* = \frac{a - bq_2 - c}{2b} = \frac{a-c}{2b} - \frac{q_2}{2}$$

This is firm 1's best response function. Given q_2, tell us that firm 1 finds it optimal to do.

What does this best response function look like?

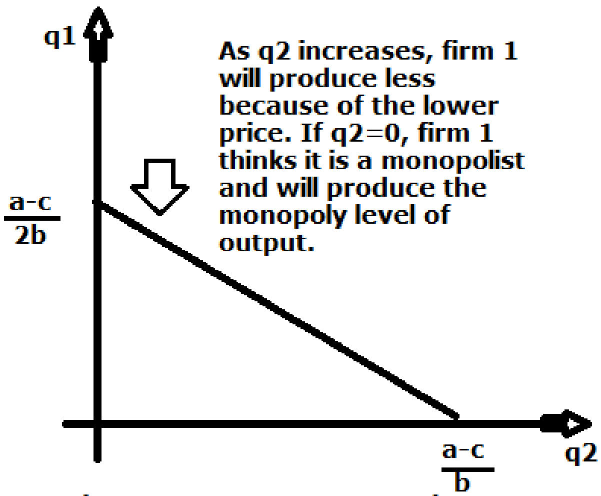

Figure 7.e1: Best response in Cournot duopoly model

As q2 increases, firm 1 will produce less because of the lower price. If q2=0, firm 1 thinks it is a monopolist and will produce the monopoly level of output. Now let's look at firm 2's problem.

$$\pi_2 = (a - bq_1 - bq_2)q_2 - cq_2$$

Note that π_2 *and* π_1 are symmetric (same function just flip the 1's and the 2's). It means that firm 2's FOC is the same.

$$q_2 = \frac{a-c}{2b} - \frac{q_1}{2}$$

Plotting firm 2's best response function yields

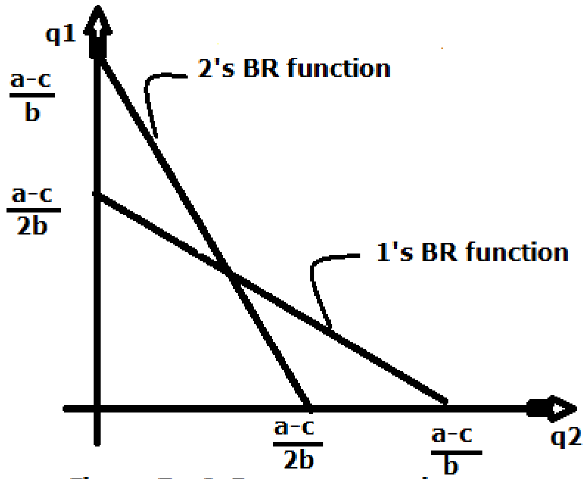

Figure 7.e 2: Best response in Cournot duopoly model

What does the equilibrium look like? It occurs at the intersection of the BR function.

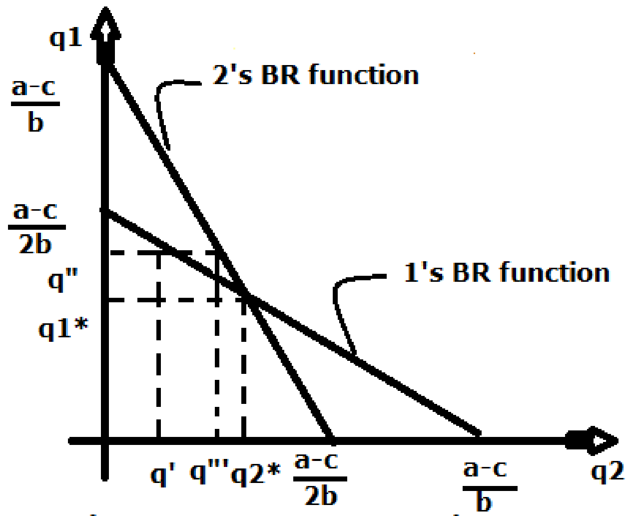

Figure 7.e3: Best response in Cournot duopoly model

If firm 1 expects 2 to produce q2*, firm 1 finds it optimal to produce q1*. If firm 2 expects firm 1 to produce q1*, firm 2 will produce q2*. So everyone is maximizing profts and expectations are consistent with actions.

Is (q1*, q2*) the unique equilibrium? Suppose firm 1 expects firm 2 to produce q'. Then firm 1 will find it optimal to produce q". But firm 1 knows that if firm 2 expected it to produce q", firm 2 would produce q"'. In which case firm 1 no longer finds it optimal to produce q". Actions and expectations are not consistent with each other.

Solving for the equilibrium?

We know that at (q1*, q2*)

- $q_1^* = \frac{a-c}{2b}$-$\frac{q_2^*}{2}$
- $q_2^* = \frac{a-c}{2b}$-$\frac{q_1^*}{2}$

Plug one equation into the other.

- $q_1^* = \frac{a-c}{2b}$-$\frac{q_2^*}{2}$
- $q_2^* = \frac{a-c}{2b}$-$\frac{q_1^*}{2}$=$\frac{a-c}{2b} - \frac{1}{2}[\frac{a-c}{2b} - \frac{q_1^*}{2}]$ ➔ one equation one unknown.
- $\frac{3q_1^*}{4} = \frac{a-c}{4b}$
- $q_1^* = \frac{a-c}{3b}, \quad q_2^* = \frac{a-c}{2b} - \frac{1}{2}\left(\frac{a-c}{3b}\right) = \frac{a-c}{3b}$
- $so \;\; Q^* = q_1^* + q_2^* = \frac{2}{3}(\frac{a-c}{b})$
- $p^* = a - bQ^* = a - \frac{2}{3}(a-c) = \frac{a}{3} + \frac{2c}{3} = \frac{a+2c}{3}$

How does this allocation compare to other market structures?
1). Under perfect competition, p=c.
Here
$p = \frac{a+2c}{3} > c \; if \; a >$
$c, which \; is \; true \; because \; otherwise \; firms \; wouldn't find \; it \; optimal \; to \; produce \; anything.$

2) Under monopoly

$$Q = \frac{a-c}{2b}, so \;\;\; p = a - bQ = a - \left(\frac{a-c}{2}\right) = \frac{a+c}{2}$$

The oligopoly price is lower if

$$\frac{a+2c}{3} < \frac{a+c}{2} \quad \Leftrightarrow 2a + 4c < 3a + 3c$$

$$\Leftrightarrow c < a$$

So we get the expected result that

1) Duopoly is not efficient (less is produced than under PC)
2) Duopoly yields fewer distortions than monopoly because more is produced.

Graphically, these have shown in figure 7.e4.

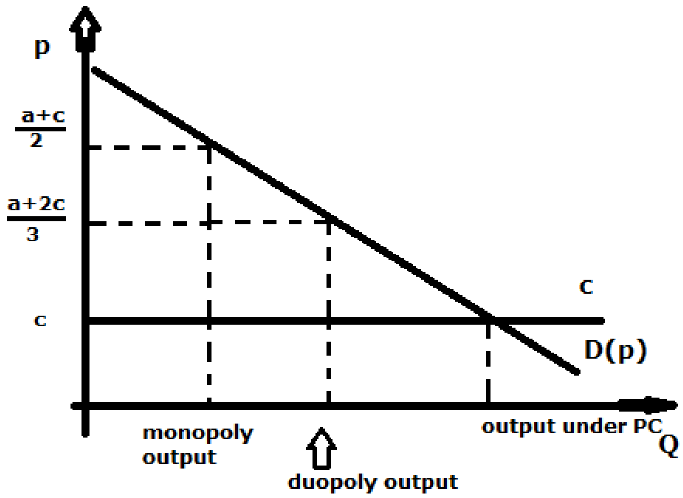

Figure 7.e4 equilibrium in monopoly, PC and Cournot duopoly

Show that $p^{PC} < p^{Cournot} < p^{Monopoly}$

Intuition:

- ➢ Higher price than PC because firms do have market power. They know that by restricting output, price will increase.
- ➢ Lower prices than monopoly because firms' consumers become more price sensitive when they can purchase from firms' competitors.
  - ✓ If firm 1 a duopolist, $\epsilon_{du} = \frac{dQ_1}{dp}\frac{p}{Q_1} = -\frac{1}{b}\frac{a-bq_1-bq_2}{q_1} = -\frac{a}{bq_1} + 1 + \frac{q_2}{q_1}$
  - ✓ If firm 1 a monopolist, $q_2 = 0 \; and \; \epsilon = -\frac{a}{bq_1} + 1$

So under Cournot, we get this intuitive result that prices are lower than under monopoly greater than under PC. But Cournot models are often criticized because

1) Firms choose prices, not quantities.
2) The equilibrium concept abstract. How does firm 1 know that 2 will produce q2* and vice versa.

Response to 1): we can re-describe our model as one ins which firms choose "Capacity constraints" or inventory level.

- ✓ Firms do not choose how much output to sell. Rather, they decide how large their factories will be. They choose their capacity q1 is equal to the largest amount of stuff that firm 1 can produce.
- ✓ After firms choose their capacity levels, they choose prices:

The most firm 1 can produce is $\frac{a-c}{3}$. The most firm 2 can produce is $\frac{a-c}{3}$.

They choose prices to maximize profits.

- ❖ Note that p1=p2. Otherwise 1 firm would sell nothing.
- ❖ They must charge a price such that they are operating at capacity. If they charge too low a price then there is excess demand. If they charge too high a price, thus is unused, wasted capacity. They will choose a price so that demand is exactly equal to q1+q2.

Response to 2):

- ✓ The Cournot equilibrium occurs at the intersection of the BR functions. Since there is only intersection, we can justify out assumption of simultaneous choices. Firms know there is only one possible equilibrium. Therefore they choose that equilibrium.
- ✓ Or we can interpret (q1*, q2*) as the result of a bargaining process:

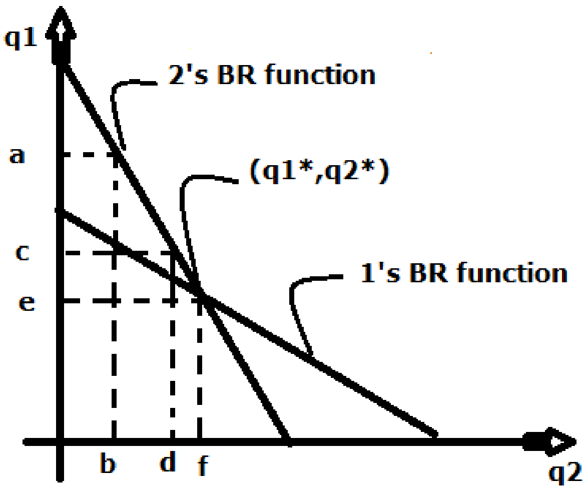

Figure 7.e5: Best response in Cournot duopoly model barging process

1) Firm 1 announces he will produce a.
2) Firm 2 says "If you produce a I will produce b."
3) Firm 1 says " if you produce b I will produce c".
4) Firm 2 says "If you produce c I will produce d".

   …..

   Eventually converge to 9q1*, q2*).

- ✓ So we can think of the Cournot equilibrium as he result of a lot of back and forth negotiations. Or we can think of as the long-run equilibrium that emerges after firms interact from many years.

Now two variations of the model:

- ➢ Asymmetric cost
- ➢ N>2 firms

Assume firm 1 has costs c1q1 and firm 2 has costs c2q2.

Now $\pi_1 = (a - bq_1 - bq_2)q_1 - c_1 q_1$

- $\frac{d\pi_1}{dq_1} = 0$ ➔ $q_1^* = \frac{a-c_1}{2b} - \frac{q_2}{2}$, $q_2^* = \frac{a-c_2}{2b} - \frac{q_1}{2}$

These best response functions are no longer symmetric, so we should not expect $q_1^* = q_2^*$.

$$q_1^* = \frac{a-c_1}{2b} - \frac{1}{2}\left(\frac{a-c_2}{2b} - \frac{q_1^*}{2}\right)$$

$$\frac{3q_1^*}{4} = \frac{a-c_1}{2b} - \frac{a-c_2}{4b} = \frac{2a-2c_1-a+c_2}{4b} = \frac{a-2c_1+c_2}{4b}$$

$$\begin{cases} q_1^* = \dfrac{a-2c_1+c_2}{3} \\ q_2^* = \dfrac{a-2c_2+c_1}{3} \end{cases}$$

Firms' production levels are decreasing in their own costs but increasing in their competition cost.

Intuition?

- 1) Costs go down, output increase for usual reasons.

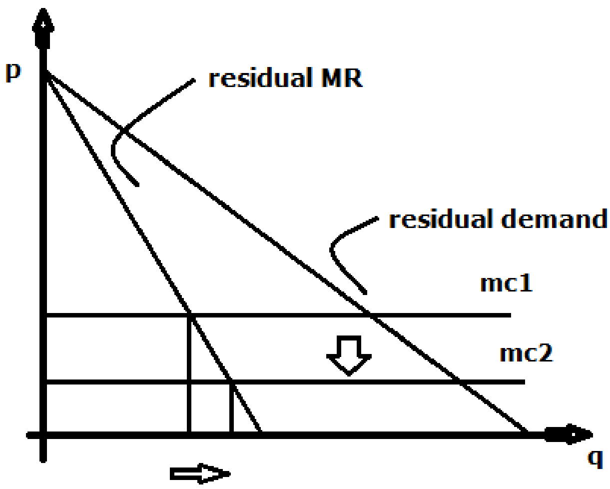

Figure 7.e 6 Asymmetric costs

Costs go down from MC1 to MC2 and you can produce more while keeping MR>=MC.

- 2) Your competitor costs increase, you produce more.
  - ✓ This effect can be seen through the best response curves.

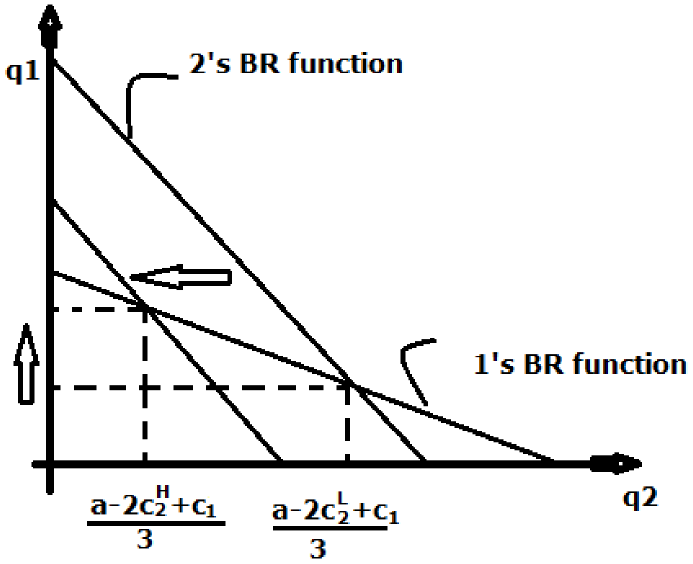

Figure 7.e7 Asymmetric costs - BR curves

Firm 2's costs go up from c_2^L to c_2^H. This causes firm 2 to be less willing to produce. BR2 shifts downward. Firm 1, knowing that firm 2 will produce less, will produce more.

- ✓ So in duopolies with asymmetric costs, the low cost firm will dominate the market.

Now let us assume N>2 identical firms.

Firm 1's profits are

$$\pi = (a - bq_1 - bq_2 - \cdots - bq_N - c = 0$$

FOC: $\frac{d\pi}{dq_1} = 0 \Leftrightarrow a - 2bq_1 - bq_2 - \cdots - bq_N - c = 0 \Leftrightarrow q_1^* = \frac{a-c}{2b} - \frac{1}{2}(q_2 + \cdots + q_N)$

Similarly, $q_k^* = \frac{a-c}{2b} - \frac{1}{2}\sum_{j \neq k} q_j$

Solving for the equilibrium? We know that all firms are identical so we should expect $q_j^* = q^*\ for\ all\ j.$ plugging this in

$$q^* = \frac{a-c}{2b} - \frac{N-1}{2}q^*$$

$$\frac{N+1}{2}q^* = \frac{a-c}{2b} \quad \rightarrow q^* = \frac{a-c}{(N+1)b}$$

If firm 1's competitor all produce $\frac{a-c}{(N+1)b}, then\ firm\ 1\ find\ it\ optimal\ to\ produce\ \frac{a-c}{(N+1)b}$

So…total output =Q=N$q^* = \left(\frac{N}{N+1}\right)\left(\frac{a-c}{b}\right)$

$$price = a - bQ^* = a - \left(\frac{N}{N+1}\right)(a-c)$$

Notice that as competition increases, (N increases,) total output increase so prices fall.

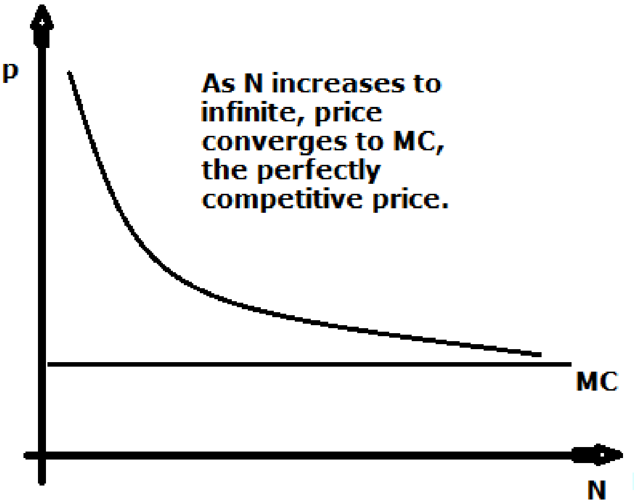

Figure 7.e8 N increases

Price ➔ c as N increases since $p^* = a - \left(\frac{N}{N+1}\right)(a-c) = a - \left(\frac{N}{N+1}\right)a + \left(\frac{N}{N+1}\right)c$

Where $a - \left(\frac{N}{N+1}\right)a$ gets close to 0 as N gets big; $\left(\frac{N}{N+1}\right)$ gets close to c as N gets big.

So the Cournot model tells us that more competition will always lead to lower prices. Intuition? As N increases, each firm's residual demand curve becomes increasingly elastic, so they have less of incentive to restrict output.