Introduction to Industrial Organization
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Lecture Note 8

Oligopolies and Collusion (ch 7 and ch. 8)

We have done all of the math necessary to solve Cournot models. Now we will study cartels within the context of a Cournot framework.

Consider a 2 firm Cournot equilibrium.

$$q_1^* = q_2^* = \frac{a-c}{3b}$$

$$p^* = \frac{a+2c}{3}$$

So profits for each firm are…

$$\pi_1 = \pi_2 = (p^* - c)q^* = \left(\frac{a-c}{3}\right)\left(\frac{a-c}{3b}\right) = \frac{1}{b}\left[\frac{a-c}{3}\right]^2$$

Total profits are equal to $\frac{2}{b}\left[\frac{a-c}{3}\right]^2 = \frac{2}{9b}(a-c)^2$

How does this compare to monopoly profits? Under Monopoly…

$Q^* = \frac{a-c}{2b}$ $p^* = \frac{a+c}{2}$ $\pi^{monop} = (p^* - c)Q^* = \left(\frac{a-c}{2}\right)\left(\frac{a-c}{2b}\right) = \frac{1}{4b}(a-c)^2$

So, monopoly profits are greater than total Cournot profits! Chapter 8 talks about how competing firms can form cartels to try to obtain monopoly profits.

Cartel: an association of firms that explicitly coordinates its pricing or output activities to collectively increase profits.

Suppose firm 1 and firm 2 from a cartel. They jointly choose q1 and q2 to maximize total profits…

$$\begin{aligned}\pi^{cartel} &= \pi_1 + \pi_2 = (p-c)q_1 + (p-c)q_2 \\ &= (a - bq_1 - bq_2 - c)(q_1 + q_2) \\ &= [a - b(q_1 + q_2) - c](q_1 + q_2) \\ &= (a - bQ - c)Q\end{aligned}$$

This profit function is identical to the monopolist's. To maximize cartel profits, choose q_1 and q_2 such that $q_1 + q_2 = \frac{a-c}{2b}$ $and\ p = \frac{a+c}{2}$ and $\pi_1 + \pi_2 = \frac{(a-c)^2}{4b}$

Does it matter how profits and output is divided between the firms? Yes. Suppose firm 1 offer firm 2 a cartel agreement. Firm 2 will only join the cartel agreement if his split of the cartel profits are greater than the Cournot profits. This is possible since

$$\pi^{cartel} > 2 \times \pi_1^{Cournot} = 2\pi_2^{Cournot}$$

This is obviously a way to split the pie so that both firms do better under cartel than Cournot.

Why does this happen? Why does colluding increase overall profits? To answer this look at the first order conditions…

Consider the cartel, first order on q1.

$$\begin{aligned}\pi^{cartel} &= (a - bq_1 - bq_2)(q_1 + q_2) - c(q_1 + q_2) \\ &= p(q_1, q_2)(q_1 + q_2) - c(q_1 + q_2)\end{aligned}$$

$$\frac{d\pi^{cartel}}{dq_1} = \frac{dp}{dq_1}(q_1 + q_2) + p(q_1, q_2) - c = 0$$

This FOC embodies the costs and benefits to the cartel from having firm 1 produce 1 extra unit of output. The benefit of one extra q1 is that the cartel collects an extra $p(q_1, q_2)$ dollars. The cost of one extra q_1 is the marginal cost plus the revenue lost when the firm has to decrease price by $\frac{dp}{dq}$ dollars in order to sell the extra unit of output. This price decrease hurts firm 1 and firm 2. When the cartel chooses q_1, it takes into account the effect that $\frac{dp}{dq_1}$ has on both firms. Notice that the FOC includes $\frac{dp}{dq_1}(q_1 + q_2)$!

In contrast, when a Cournot firm chooses q_1, he does not take into account how his output choice affects firm 2.

$$\pi_1^{Cournot} = p(q_1, q_2)q_1 - cq_1$$

$$\frac{d\pi_1^{Cournot}}{dq_1} = \frac{dp}{dq_1}q_1 + p - c = 0$$

Where $\frac{dp}{dq_1}q_1$ shows that the Cournot firm only internalizes how price changes affect him.

Conclusion: Cartel are able to increase profits for everyone because they force participating firms to internalize how their output choices affect other firms.

What determines the existence of cartels?

- 1) it needs to be profitable to form the cartel
- 2) the cartel needs to be sustainable.

When is it profitable to form cartels?

- Demand must be inelastic. If demand is elastic, firms cannot increase prices even if they do collude.
- The cartel must have a large share of the market. Cannot be close substitutes to

cartel's profits.

- Legal consequences of getting caught must be low. Cartels were much more common in the US before 1890 when collusion was made illegal.
- Low organizational costs. It must be easy to organize the cartel.
  - ✓ Thus cannot be too many firms involved
  - ✓ Products must be similar
  - ✓ It must be easy for colluding firms to communicate with each other

When are cartels sustainable?
It is hard to sustain cartels because firms have incentives to cheat.

Consider the two firm cartel.

$$q_1^{cartel} = q_2^{cartel} = \frac{1}{2}Q^{monop} = \frac{a-c}{4b}$$

$$\pi_1^{cartel} = \pi_2^{cartel} = \frac{1}{2}\pi^{monop} = \frac{(a-c)^2}{8b}$$

If firm 1 expects firm 2 to keep producing q_2^{cartel}, is it profit maximizing for firm 1 to keep producing q_1^{cartel}?

$$\pi_1^{cheat} = \left(a - 2bq_1 - bq_2^{cartel}\right)q_1 - cq_1$$

$$\frac{d\pi_1^{cheat}}{dq_1} = a - 2bq_1 - bq_2^{cartel} - c = 0$$

$$q_1^{cheat} = \frac{a-c}{2b} - \frac{q_2^{cheat}}{2} = \frac{3(a-c)}{8b} > q_1^{cartel}!$$

Firm 1 has an incentive to over produce in order to increase his share of the profits.

Now, $Q = q_1^{cheat} + q_2^{cartel} = \frac{5(a-c)}{8b}$ so $p = \frac{3a+5c}{8}$

And $\pi_1^{cheat} = (p-c)q_1^{cheat}$

$$= \left(\frac{3a+5c}{8} - c\right)\left(\frac{3a-3c}{8b}\right) = \frac{1}{b}\left(\frac{3a-3c}{8}\right)^2 = \frac{9(a-c)^2}{64b}$$

Note that $\pi_1^{cheat} > \pi_1^{cartel}$! This result comes straight from firm 1's best response function.

Figure 7.e9 illustrates that if firm 1 chooses to ignore how his behavior affects firm 2's price, he finds it optimal to produce q_1^{cheat}.

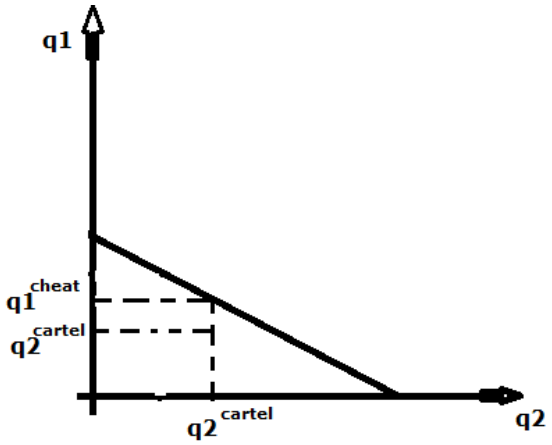

Figure 7.e9 Cartel

Conclusion:

- We will see that in cartels, there is always an incentive to cheat.
- Some markets have exogenous, characteristics that make cheating hard:
  - ✓ 1) few firms in the market
  - ✓ 2) there are not random shakes to prices/quantities
  - ✓ 3) prices/quantities are observable
  - ✓ 4) certain cost structures make cheating less attractive
- Furthermore, cartel agreements can be constructed to make cheating unprofitable. For example, firm 1 can tell firm 2 it will start a price war if cheating is ever observed. We will study these types of enforcement mechanisms using multi-period models of oligopoly.
- We will make two points about sustaining cartels using multi-period games.
  - ✓ It is easier to sustain cooperation when firms expect to interact over long periods of time.
  - ✓ Firms decisions about whether to cooperate depends on their beliefs about that will happen if they stop cooperating.

First lets consider a world in which firms compete against each other for one or two periods. Some setup as before.

Demand: p=a-bQ
Costs: TC=cq
Model: Simultaneous move Cournot with restricted choices. Each firm simultaneously decide whether to produce the low, cartel level of output $q^L = \frac{a-c}{4b}$ or the high, Cournot level of output $q^H = \frac{a-c}{3b}$.

We know what will happen if both firms produce q^L or if both firms produce q^H :

- If both firm produce q^H:
  - ✓ Price will be $\frac{a+2c}{3}$
  - ✓ Both firms will earn profits $\pi = \frac{1}{9b}(a-c)^2$
- If both firm produce q^L:
  - ✓ Price will be $\frac{a+c}{2}$
  - ✓ Both firms will earn profit $\pi = \frac{1}{8b}(a-c)^2$

What if firm 1 produces q^L and firm 2 produces q^H (i.e. firm 2 cheats)?

$$p = a - b(q^L + q^H) = a - b\left(\frac{7a-7c}{12b}\right) = \frac{5a+7c}{12}$$
$$\pi_1 = (p-c)q^L = \left(\frac{5a-5c}{12}\right)\left(\frac{a-c}{4b}\right) = \frac{5}{486}(a-c)^2$$
$$\pi_2 = (p-c)q^H = \left(\frac{5a-5c}{12}\right)\left(\frac{a-c}{3b}\right) = \frac{5}{366}(a-c)^2$$

Assume that $\frac{(a-c)^2}{b} = 1$. then per period profits given by:

Firm 1 \ Firm 2	q_L	q_H
q_L	$\frac{1}{8}, \frac{1}{8}$	$\frac{5}{48}, \frac{5}{36}$
q_H	$\frac{5}{36}, \frac{5}{48}$	$\frac{1}{9}, \frac{1}{9}$

Figure 7.e10 Profits given any output choices

Note that $\frac{5}{48} < \frac{1}{9} < \frac{1}{8} < \frac{5}{36}$.

What is the equilibrium if firms compete for only one period?

Looking for (q_1^*, q_2^*) such that:

- 1) q_1^* is optimal given that firm 2 chooses q_2^*
- 2) q_2^* is optimal given that firm 1 chooses q_1^*.

Easy to show that collusion cannot be sustained:

- If $q_2=q_L$, firm 1 is better off at q_H
- If $q_1= q_L$, firm 2 is better off at q_H

⇨ When firms interact only once, collusion is impossible!

What if firm 1 says to firm 2 in period 1 " I will never produce q_H as long as I expect you to produce q_H". Firm 2 says to firm 1 the exact same thing.

Is it rational for the firms to expect each other to produce the cartel output levels? If so, the collusion can be sustained.

Unfortunately, it is not rational to expect firms to not cheat whenever there are a finite number of periods.

Suppose that both firms produce q_L in period 1. What then happens in period 2? In period 2, after period 1 is over, the game firms play against each other becomes just like the one period game already discussed. The period 2 equilibrium is q^H, q^H.

So now in period 1, both firms know that no matter what they will do in period 1, in period 2 the equilibrium will be q^H, q^H. Knowing this, both firms have incentives to cheat in period 1. In period 1 firm will both produce q^H. So unless firms interact for a very long time, collusion impossible!!

The logic behind the 2 period model can be extended to any model with N less than infinite periods. The collusive agreement will not happen. Next we will consider games in which firms interact indefinitely and can punish each other.

Assume the same setup before, but firms interact forever. So if both firms collude forever, they earn profits equal to $\pi^{collude} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \cdots$

If they compete forever they earn profits $\pi^{compete} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \cdots$

We will now show that collusion can be sustained if $N = \infty$ and firms can punish each other.

Suppose that firm 1 says to firm 2, " I will produce q^L each period. But if I observe you produce q^H even once, I will then produce q^H until I have observed you producing q^L for T period in a row."

If firm 2 believes firm 1's threat, what should firm 2 do?

If firm 2 does not cheat….