

**Introduction to Industrial Organization**  
**Professor: Caixia Shen Fall 2014**  
**Lecture Note 9**

**Oligopolies and Collusion (ch 7 and ch. 8) – continue**

Assume that the same set up before, but firms interact forever. So if both firms collude forever, they earn profits equal to  $\pi^{collude} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \dots$

If they compete forever, they earn profit  $\pi^{compete} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \dots$

We will now show that collusion can be sustained if  $N = \infty$  and firms can punish each other.

Suppose that firm 1 says to firm 2, “ I will produce  $q^L$  each period. But if I observe you produce  $q^H$  even once, I will then produce  $q^H$  until I have observed you producing  $q^L$  for T period in a row.”

If firm 2 believes firm 1’s threat, what should firm 2 do?

If firm 2 does not cheat....

$$q_2 = q_L, q_L, q_L, \dots$$

$$q_1 = q_L, q_L, q_L, \dots$$

And

$$\pi_2^{no\ cheat} = \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \dots$$

If firm 2 cheats for one period...

$$q_2 = q_H, q_H, q_H, \dots, q_H, q_L$$

$$q_1 = q_L, \underbrace{q_H, q_H, \dots, q_H}_{T\ periods}, q_L$$

And

$$\pi_2^{cheat} = \frac{5}{36}, \frac{1}{9}, \frac{1}{9}, \dots, \frac{1}{9}, \frac{1}{8}, \frac{1}{8}$$

T periods

So firm 2 will not cheat as long as

$$\pi_2^{no\ cheat} > \pi_2^{cheat}$$

↔

$$\pi_2^{no\ cheat} - \pi_2^{cheat} > 0$$

↔

$$\underbrace{\left[\left(\frac{5}{36} - \frac{1}{8}\right)\right]}_{\text{gain from cheating}} - \underbrace{\left[\left(\frac{1}{8} - \frac{1}{9}\right) + \left(\frac{1}{8} - \frac{1}{9}\right) + \dots + \left(\frac{1}{8} - \frac{1}{9}\right)\right]}_{\text{punishment from cheating}} < 0$$



$$T\left(\frac{1}{8} - \frac{1}{9}\right) > \left(\frac{5}{36} - \frac{1}{8}\right)$$

*this will be true as long as the punishment is sufficiently long*

Should firm 1 threaten this punishment?

Yes! Because if the threat is made, firm 1 will earn 1/8 forever, which is better than 1/9 forever. Similarly, if firm 2 makes a similar threat to firm 1, firm 1 will never cheat on firm 2. The firms will collude forever.

Note that this model predicts that cheating will never occur. But punishments (price war) are observed in the real world. For example, automobile production in the 1950's was low except for one year:

Domestic automobile production:

1953	6.1 million
1954	5.5 million
1955	7.9 million
1956	5.8 million
1957	6.1 million
1958	4.2 million
1959	5.6 million

In all years except 1955, production was low and prices were high. In 1955 output shot up and prices fell by 6%. This was interpreted as a punishment after colluding firms mistakenly detected cheating.

So if firms mistakenly detect cheating, collusive agreements can fall apart. In our model, we assumed that firms perfectly observe that other firms are doing. This is not true in reality. And the reason for why collusion is more likely to occur in markets with few firms, without random shocks to output levels, and then things are relatively easy to observe.

Note that our model predicts cheating will never occur but price wars are observed in the real world: the automobile production in the 1950's....

How do we amend our model to generate collusion and price wars? Suppose now that firms have beliefs about what market structures will look like in the future. These beliefs can be embodied into a discount factor  $\delta$ , in a general way.

Suppose that two drug companies are colluding. There is probability  $(1 - \delta)$  that in the future a third drug company introduces a better drug eliminating the original two companies'

market.

If there is a  $1 - \delta$  probability that an improved drug is introduced, then there is a  $\delta$  that probability that the firm earns profits next period, a  $\delta^2$  probability that the firms earn profits two periods from now, and so on.

Now, if 2 cheats,

$$\pi = \pi_H + \delta\pi_L + \delta^2\pi_L + \dots + \delta^T\pi_L$$

If 2 does not cheat,

$$\pi = \pi_M + \delta\pi_M + \delta^2\pi_M + \dots + \delta^T\pi_M$$

Collude if

$$\pi_M + \delta\pi_M + \delta^2\pi_M + \dots + \delta^T\pi_M > \pi_H + \delta\pi_L + \delta^2\pi_L + \dots + \delta^T\pi_L$$

$\Leftrightarrow$

$$\pi_H - \pi_M < \delta(\pi_M - \pi_L) + \delta^2(\pi_M - \pi_L) + \dots + \delta^T(\pi_M - \pi_L)$$

$\Leftrightarrow$

$$\pi_H - \pi_M < \left( \frac{\delta - \delta^{T+1}}{1 - \delta} \right) (\pi_M - \pi_L)$$

$\left( \frac{\delta - \delta^{T+1}}{1 - \delta} \right) (\pi_M - \pi_L) \rightarrow$  the cost of cheating still increases in T, but now decreases in  $\delta$ .

Suppose firms'  $\delta$ 's are hidden from each other and they are changing over time. Then collusive agreements can fall apart if shocks to  $\delta$  give firms incentives to cheat because collusion made less attractive.

- Firm 2 might decide to start cheating if they suddenly become convinced that entry is more likely to occur in the future
- Similarly,  $\delta$  can be interpreted as future market growth.  $\delta$  increases as expected growth increases. If firm 2 becomes pessimistic about future growth, they might cheat now while the market is big.
- Or perhaps firm 2 has private information about its financial condition. If firm 2 is in bad financial condition they might place more weight on today's profits than tomorrow's profits. So, if firm 2 desperately needs money today, this can be thought of as a decrease in  $\delta$ .
- Above described how price wars can be generated by shocks to firms' preferences over today's and future profits. Price wars can also result because of incomplete information.
- In our model, price wars occur when firm 1 observes firm 2 cheating (or vice versa). But what if firm 1 never actually observes firm 2's prices or quantities. For example, in many markets firms negotiate prices directly with customers. Prices are never posted anywhere (for example negotiations between large employer and health insurance over benefits and premiums). Suppose instead that firms only observe their

own profits. If they earn 0, they have been cheated on, if they earn  $\pi_M$  they have not. What if firms profits determined by more than competitors' behavior:

$$\pi \begin{cases} \pi_M + \varepsilon & \text{if competitor colludes} \\ \varepsilon & \text{if competitor cheats} \end{cases}$$

$\varepsilon$  embodies unobserved shocks to demand

If  $\varepsilon < -\pi_M$ , a firm might confuse a demand shock with cheating and initiate a price war!

**Collusion and public policy? Testing for collusion?**

- Much of antitrust and competition policy aimed at preventing firms from “conspiring” against the consumer.
- Surplus affects of collusion are obvious.

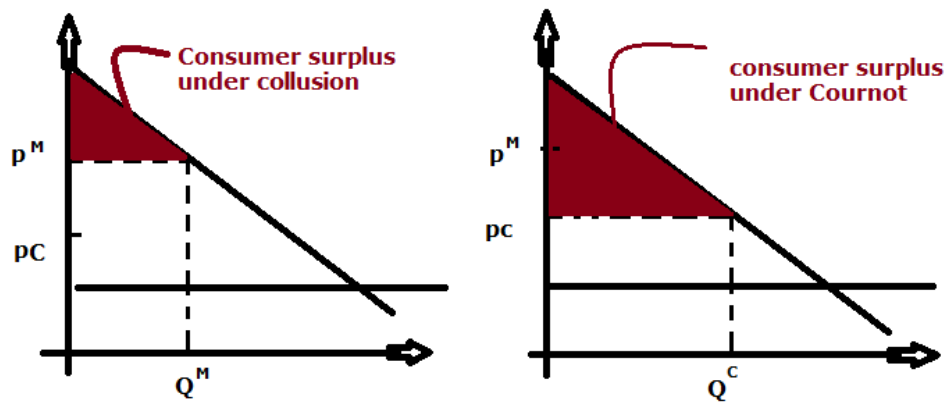


Figure 7.e11 Consumer surplus under collusion and Cournot

- When firms collude, something as monopolizing a market. Consumer surplus decreases. Total surplus decreases also. So government should prohibit collusion for efficiency reasons.
- In the US, collusion made illegal by the Sherman Act in 1890:

“Every person who shall monopolize, or attempt to monopolize, or combine or conspire with any other person or persons, to monopolize any part of trade or commerce shall be deemed guilty of a felony.”

- Virtually all other countries have similar laws, though the enforcement tends to be more rigorous in the United States.

**Now briefly talk about testing for collusion.**

Suppose we observe demand and costs. Then equilibrium price depends on the assumed model of competition. Give  $Q^D(p)$  and  $TC(Q)$ , models of competition and collusion yield different predictions about prices. We can exploit these differences in

predictions to learn about the nature of competition.

$$p = a - bQ \quad \text{and} \quad TC = cQ$$

➤ If N firms competing, then  $p = a - \left(\frac{N}{N+1}\right)(a - c)$

If N firms colluding, then  $p = \frac{a+c}{2}$

➤ Consider changes to “a”, “c”, and prices:

	<i>under collusion</i>	<i>under Cournot competition</i>
$a \uparrow \text{ by } 1$	$p \uparrow \text{ by } \$0.5$	$p \uparrow \text{ by } \frac{1}{N+1}$
$b \uparrow \text{ by } 1$	$p \uparrow \text{ by } \$0.5$	$p \uparrow \text{ by } \frac{1}{N+1}$

➤ If we observe changes in “a” and “c”, we can examine price changes to test for collusion:

$$\text{Suppose } c = \gamma(\text{oil prices}) + \omega$$

We can examine relationship between oil prices and equilibrium prices to test for collusion.

$$\text{Suppose } a = \gamma(\text{market income}) + \omega$$

We can examine relationship between prices and consumers income levels to test for collusion. For example, Aviv Nevo cereal prices.

Done with Cournot model and Collusion. Briefly do variant of Cournot model (Stackelberg model) and then Bertrand

- In the Cournot mode, we assumed that firms with identical cost cuntions behave identically because they move simultaneously and face the same demand function. In reality, firms may behave differently because:
  - ✓ 1) their costs are different
  - ✓ 2) they face different demands (i.e., thus products are slightly different. We will do this )
  - ✓ 3) firms may have advantages/ disadvantages for historical reasons (For example, apple)
- If we went to give firms advantages without changing their cost or demand functions, we can do so using Stackelburg model. This is similar to Cournot but we allow a subset of firms to make their decisions before everyone else. Similar to “competitive fringe model”, but everyone has some market power.

Set up of Model:

$$\text{Demand: } p = a - bQ$$

$$\text{Costs: } TC = cq$$

1 dominant firm (Apple) moves first  
 N follows best respond to Apple’s output choice

We will see that allowing Apple to move first gives it an advantage. Suppose that N=1. Then Apple knows what the Fringe’s best response function looks like:

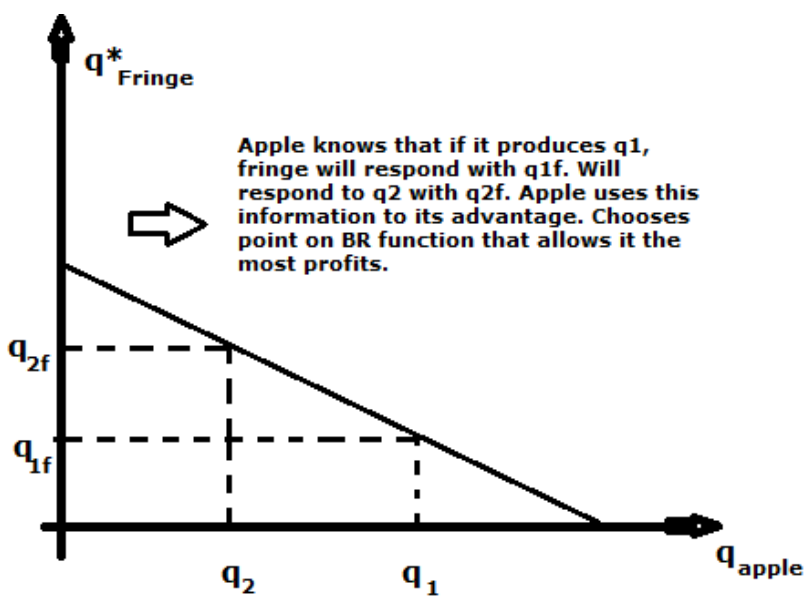


Figure 7.e12 Stackelberg model- case of Apple

Solving model:

1. Given  $q_{Apple}$ , the  $N$  fringe firms simultaneously choose  $q$ . Consider firm 1's problem:

$$\begin{aligned}\pi_1 &= (a - b[q_{apple} - q_2 - \dots - q_N])q_1 - cq_1 \\ \frac{d\pi_1}{dq_1} &= a - 2bq_1 - b[q_{apple} + q_2 + \dots + q_N] - c = 0 \\ q_1^* &= \frac{a - c}{2b} - \frac{1}{2}(q_{apple} + q_2 + \dots + q_N)\end{aligned}$$

Notice that these firms have market power.

2. Since the fringe firms move simultaneously and are identical, we can exploit symmetry.

$$\begin{aligned}q_1^* &= q_2^* = \dots = q_N^* \equiv q_f^* \\ q_f^* &= \frac{a - c}{2b} - \frac{1}{2}(q_{apple} + (N - 1)q_f^*) \\ \left(\frac{N + 1}{2}\right)q_f^* &= \frac{a - c}{2b} - \frac{q_{apple}}{2}\end{aligned}$$

3. Solve apple's profit maximization problem:

- Apple know that whatever it does, each fringe firm will produce

$$\frac{a - c}{(N + 1)b} - \frac{q_{apple}}{N + 1}$$

$$\begin{aligned}\text{Therefore } Q &= q_{apple} + Nq_f^* \\ &= q_{apple} + \left(\frac{N}{N + 1}\right)\left[\frac{a - c}{b} - q_{apple}\right] \\ &= \frac{q_{apple}}{N + 1} + \left(\frac{N}{N + 1}\right)\left(\frac{a - c}{b}\right)\end{aligned}$$

$$\pi_{apple} = \left(a - b(q_{apple} + Nq_f^*)\right)q_{apple} - cq_{apple}$$

Apple's first order condition takes into account how apple's behavior affects follower's behavior.

$$\pi_{apple} = \left(a - b\left[\frac{q_{apple}}{N + 1} + \frac{N}{N + 1}\left(\frac{a - c}{b}\right)\right]\right)q_{apple} - cq_{apple}$$

$$\frac{d\pi_{apple}}{dq_{apple}} = a - \left(\frac{N}{N + 1}\right)(a - c) - \frac{2bq_{apple}}{N + 1} - c = 0$$

$$\frac{a + Nc - (N + 1)c}{N + 1} = \frac{2bq_{apple}}{N + 1}$$

$$q_{apple}^* = \frac{a - c}{2b}$$

If  $q_{apple} = \frac{a-c}{2b}$ , then  $q_f^* = \frac{a-c}{2b(N+1)}$  and  $Q = \left(1 + \frac{N}{N+1}\right) \left(\frac{a-c}{2b}\right) = \left(\frac{2N+1}{N+1}\right) \left(\frac{a-c}{2b}\right)$

Notice that Apple's market share is equal to  $\frac{q_{apple}}{Q} = \frac{N+1}{2N+1} > 50\%$ .

Algebra shows that price decreases in  $N$  as expected, Apple earns the most profits, and total output is greater than in a Cournot equilibrium with  $N+1$  firms.